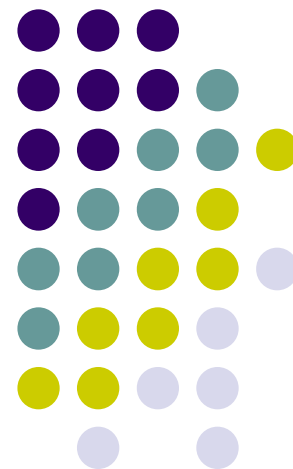


信号与系统

SIGNALS AND SYSTEMS

信息工程学院 张黎





一、教材

信号与线性系统分析 吴大正 主编(第四版)

二、参考书

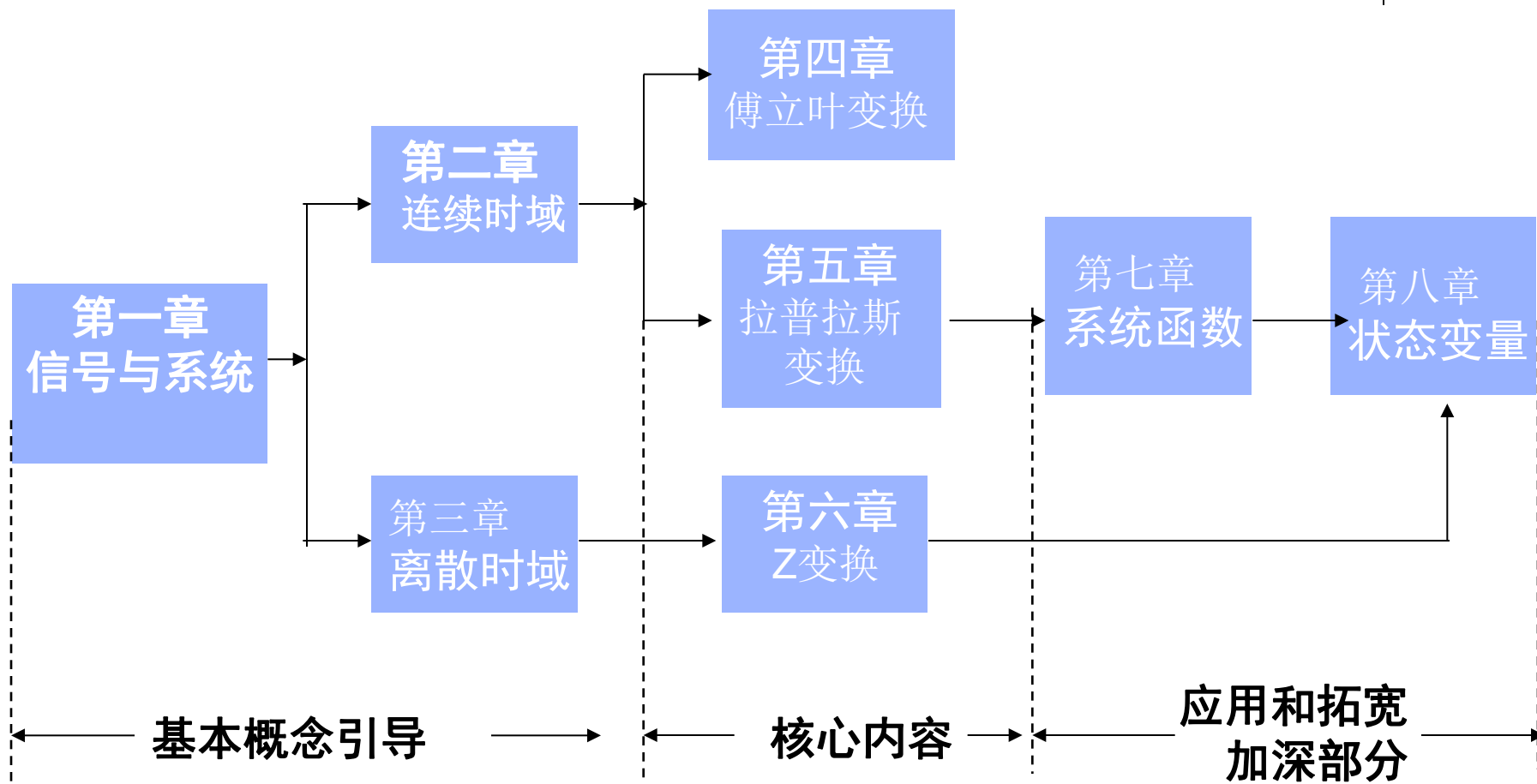
Signals & Systems (second edition) A. V. Oppenheim

信号与系统	郑君里等	高等教育出版社
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信号与系统	曾禹村等	北京理工大出版社
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信号与系统分析	罗永光等	国防科大出版社
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三、学习的主要内容



第一章 信号与系统



要点:

1. 信号的运算 (+, -, •, 自变量变换)
2. 阶跃函数 $\varepsilon(t)$, 冲激函数 $\delta(t)$
定义及其性质。
3. 线性非时变系统的性质
4. 系统的描述及框图模拟

§ 1.1 绪言 § 1.2 信号 § 1.3 信号的基本运算

一.信号(Signal)

函数 (function)

传送消息的符号或载体(物理量)

二. 系统 (System)

1. 若干相互联系,相互作用的事物(部件)组合而成的,具有某种特定功能的整体.
2. 信号系统: 信号的产生,传输,处理,存储,转换.
3. 系统的简化框图

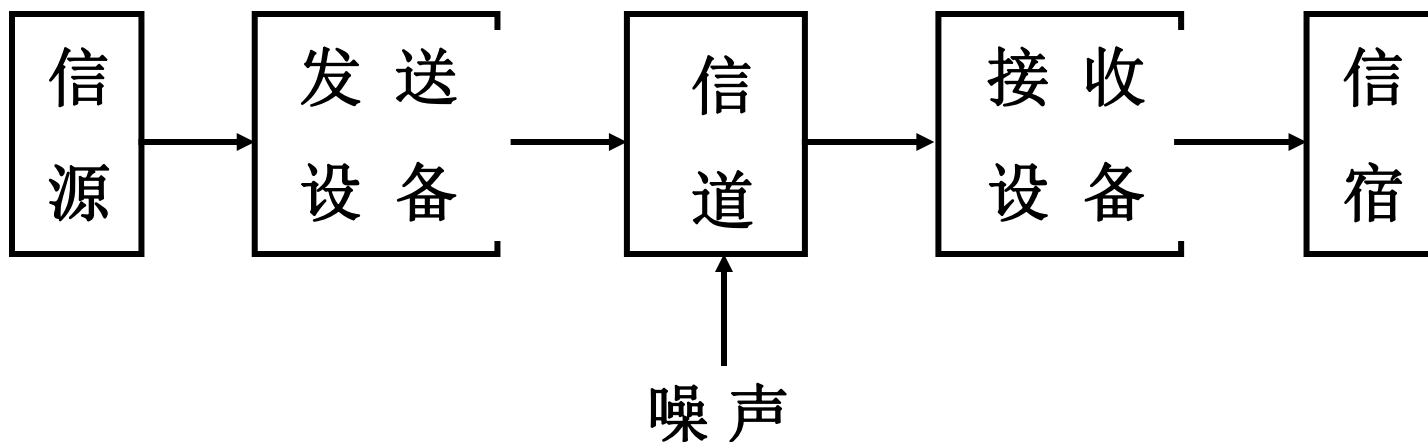


$$f(t) \longrightarrow y(t)$$

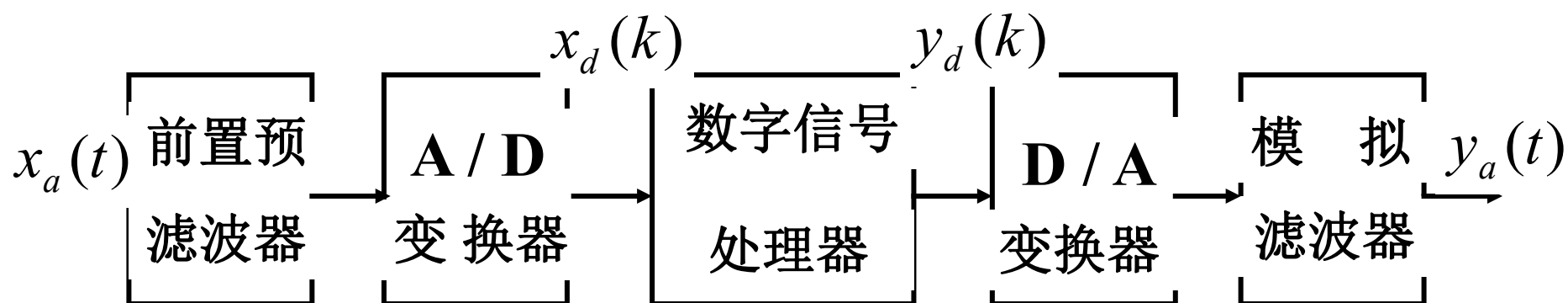
$$y(t) = T[f(t)]$$

$$f(k) \longrightarrow y(k)$$

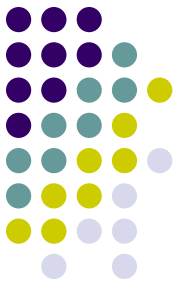
$$y(k) = T[f(k)]$$



通信系统框图



音响处理系统框图



三. 信号的分类

1. 连续信号：定义域连续 $\cos(\beta t), \varepsilon(t), e^{-\alpha t}$

离散信号：定义域离散 $\cos(\beta k), \varepsilon(k), a^k$ 

模拟信号：定义域连续，值域连续

数字信号：定义域离散，值域离散

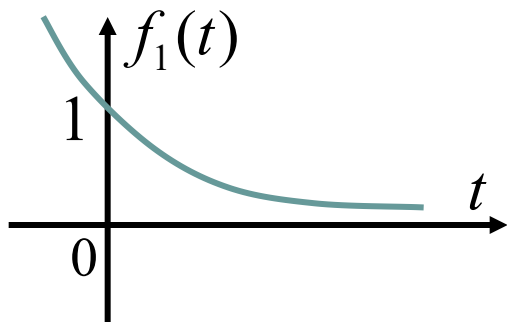
2. 实信号(实自变量t,k的实函数)

$$\cos(\beta t), \varepsilon(t), e^{-\alpha t}, \delta(t)$$

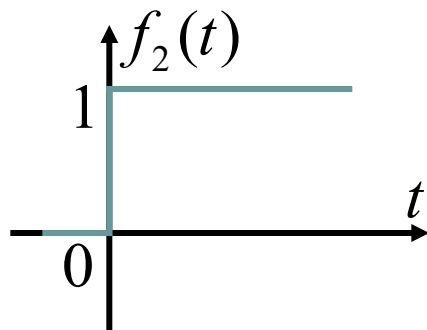
$$\cos(\beta k), \varepsilon(k), a^k, \delta(k)$$

复信号(实自变量t,k的复函数)

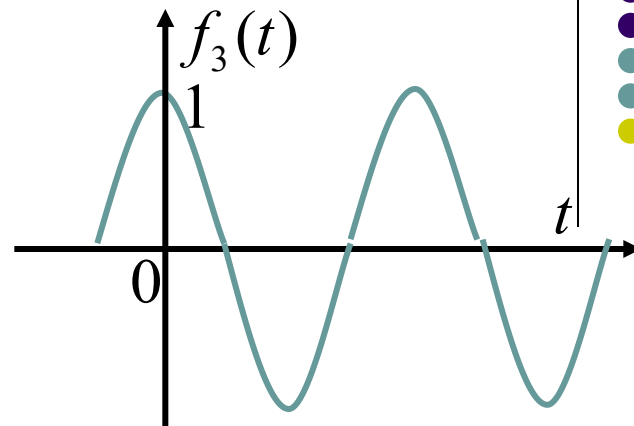
$$e^{st}, s = \sigma + j\omega, e^{j\omega t}, z^k, z = \rho e^{j\theta}$$



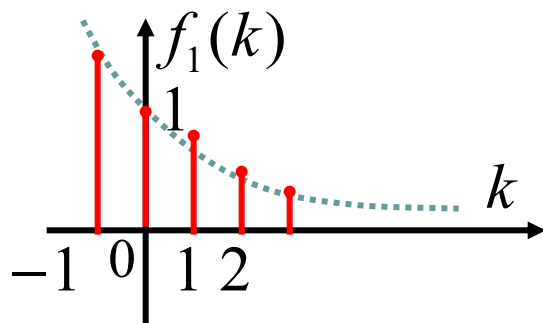
$$f_1(t) = e^{-\alpha t}$$



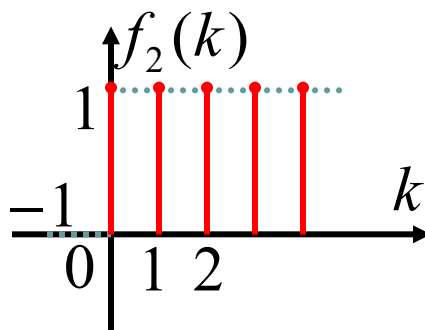
$$f_2(t) = \epsilon(t)$$



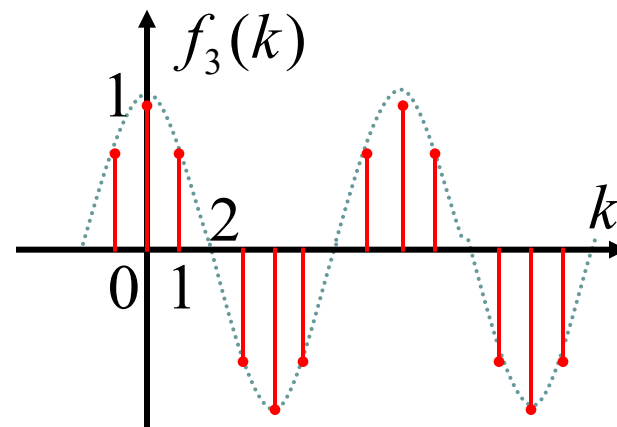
$$f_3(t) = \cos(\omega t)$$



$$f_1(k) = a^k$$

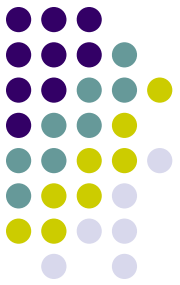


$$f_2(k) = \epsilon(k)$$



$$f_3(k) = \cos(\beta k)$$





3. 周期信号:

$$f(t) = f(t + T), \forall t, \quad \cos(\omega t), \dots$$

$$f(t) = f(t + mT), m = \pm 1, \pm 2, \dots$$

$$f(k) = f(k + N), \forall k, \quad \cos(\beta k), *$$

$$f(k) = f(k + mN), m = \pm 1, \pm 2, \dots$$

非周期信号

4. 一维信号 多维信号

四. 信号的基本运算



1. 两个信号的加,减,乘运算

$$f_1(t) \pm, \times f_2(t), f_1(k) \pm, \times f_2(k)$$



2. 自变量变换 *Transformation of the independent variable*

(1) 平移: $f(t) \rightarrow f(t \pm t_d), (t_d > 0)$

Time shift $f(k) \rightarrow f(k \pm m), (m > 0)$

(2) 反折: $f(t) \rightarrow f(-t)$

Time reversal $f(k) \rightarrow f(-k)$

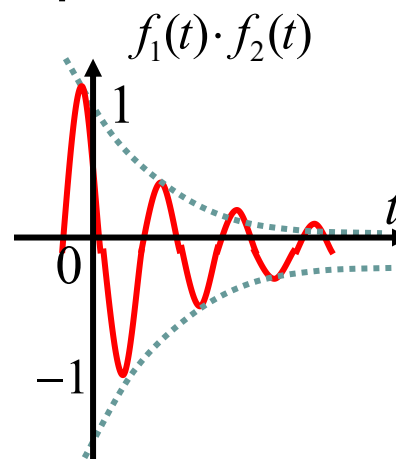
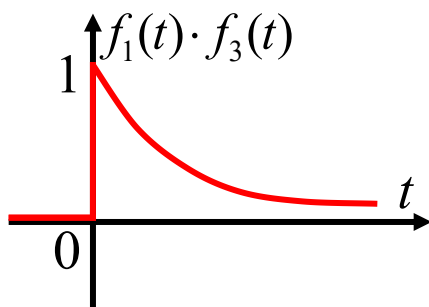
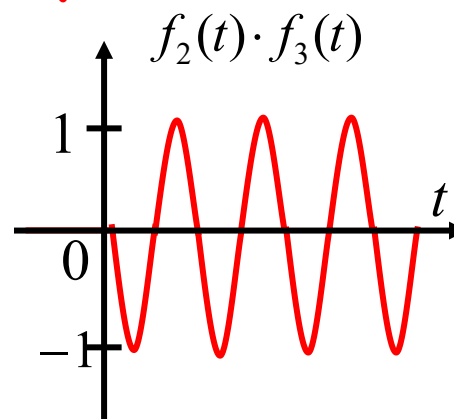
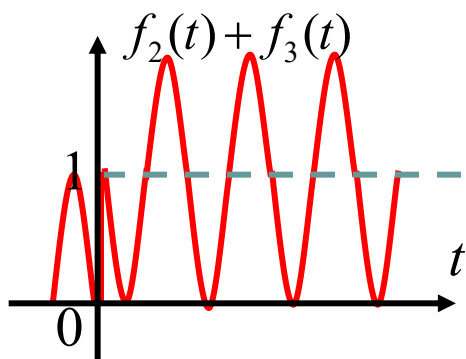
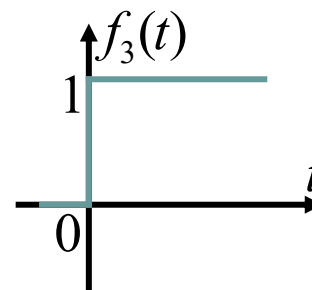
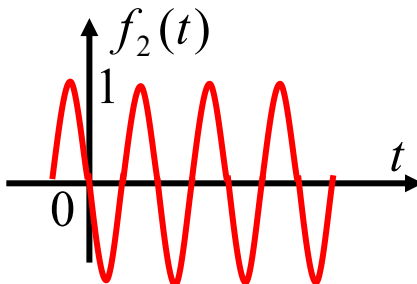
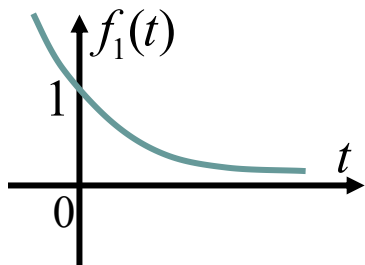
(3) 时间尺度变换: $f(t) \rightarrow f(at) \begin{cases} a > 1 \\ 1 > a > 0 \end{cases}$

Time scaling

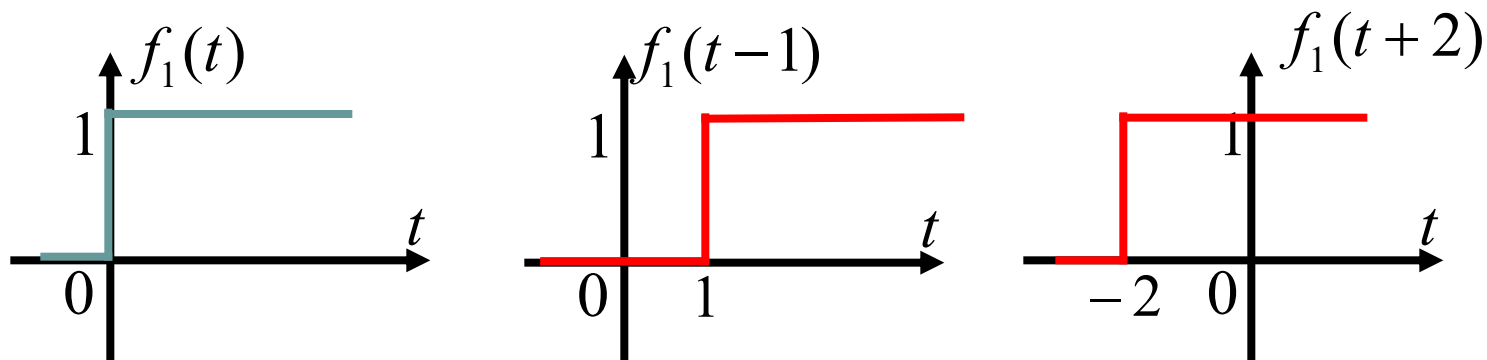
$$f(k) \rightarrow f(ak) \begin{cases} a > 1 \\ 1 > a > 0 \end{cases}$$



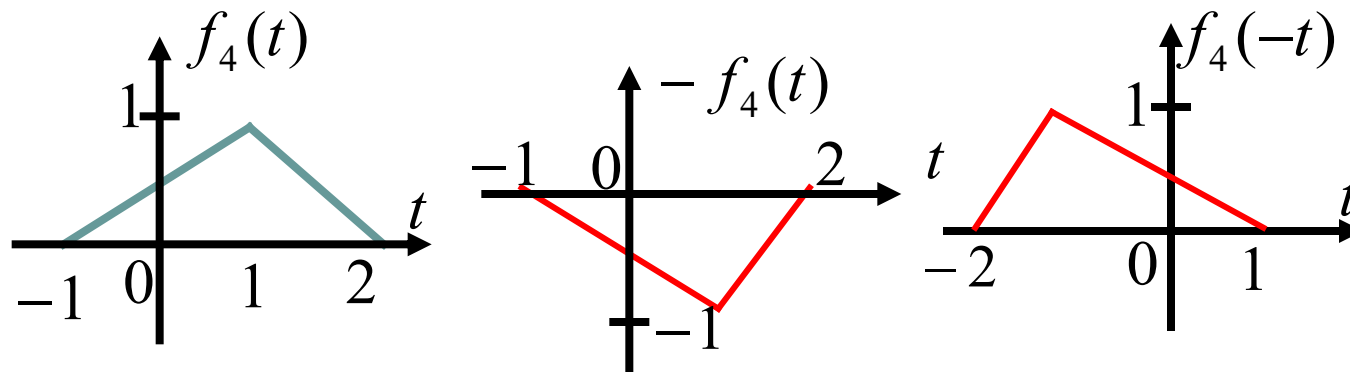
例 1.1-1



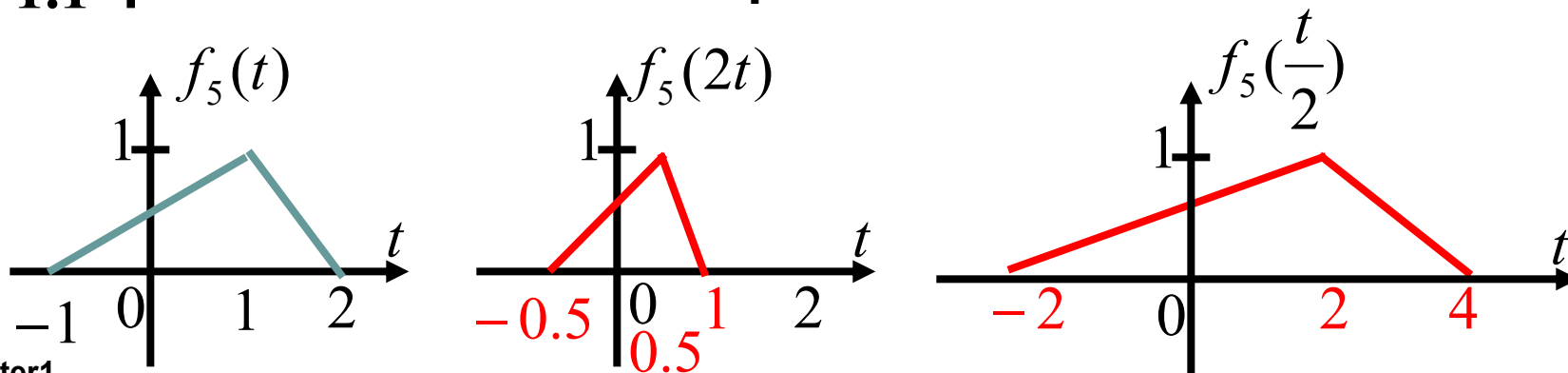
例 1.1-2



例 1.1-3



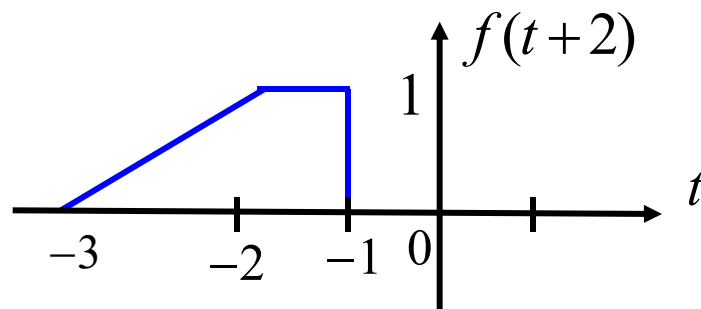
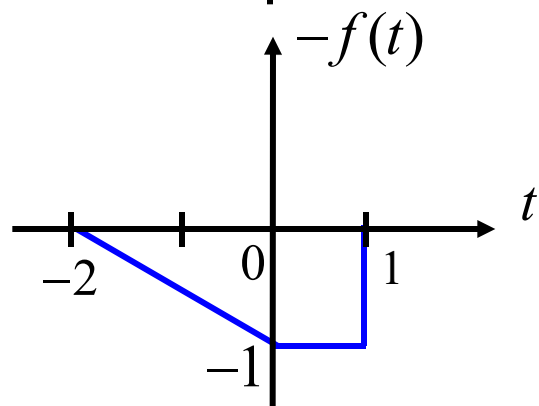
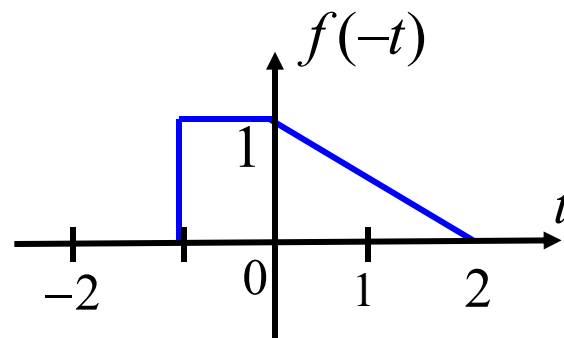
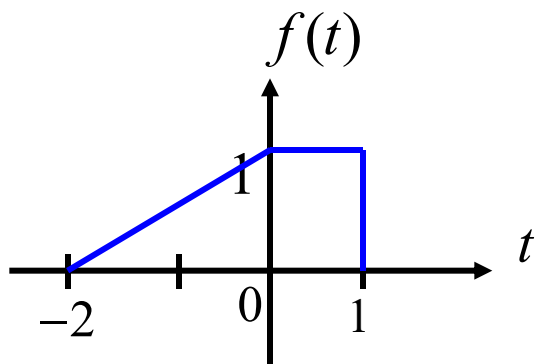
例 1.1-4

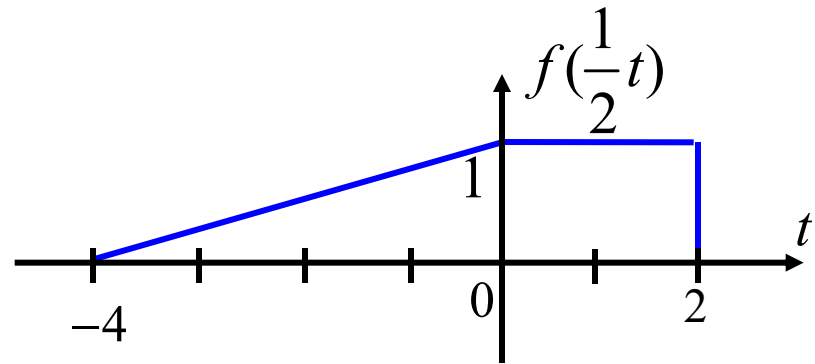
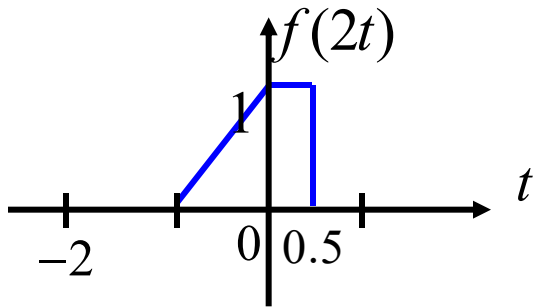
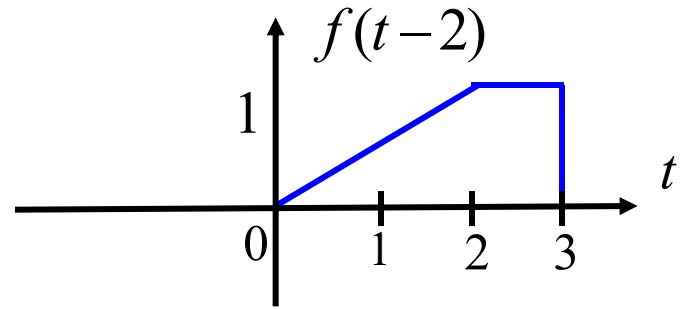
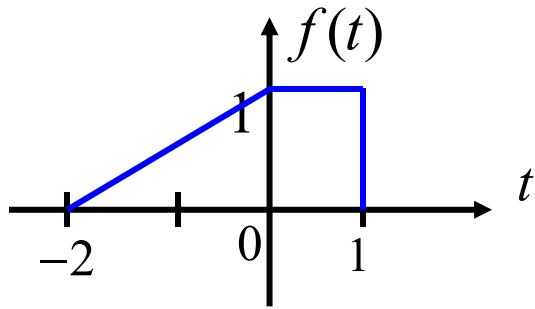
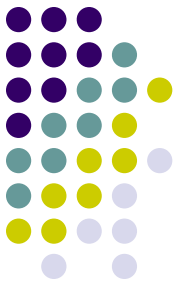




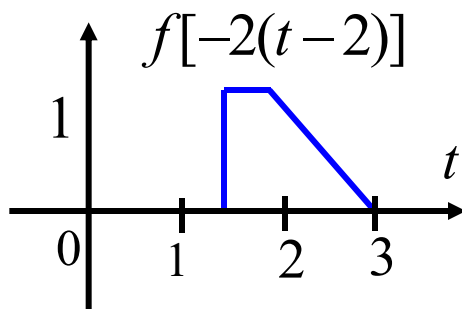
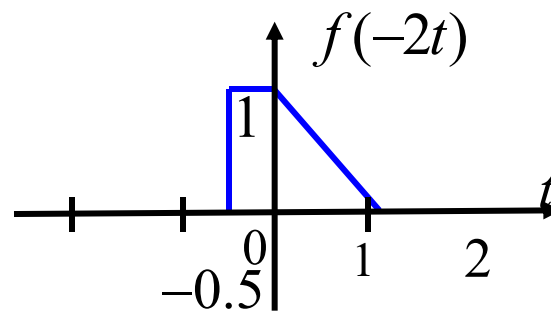
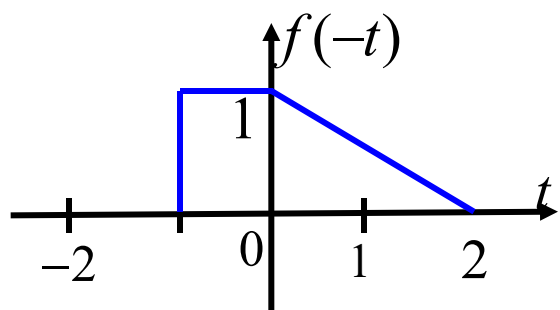
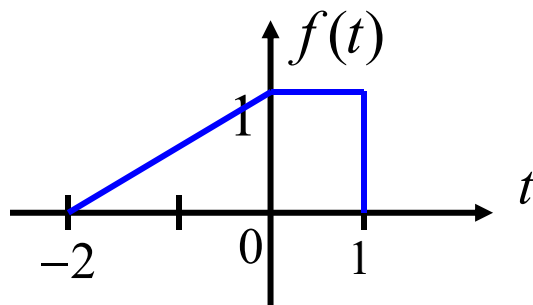
例 1.1-5 已知 $f(t)$ 波形如下，求

$$f(-t), -f(t), f(t+2), f(t-2), f(2t), f\left(\frac{1}{2}t\right), f(-2t+4)$$

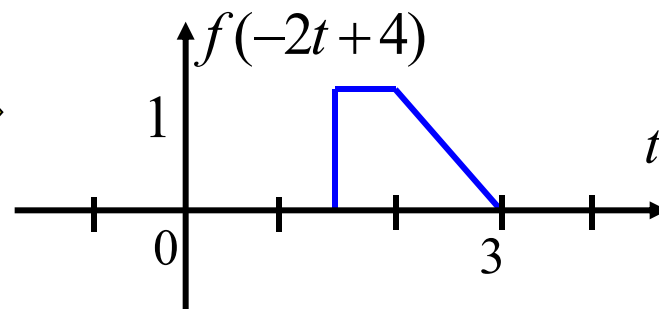
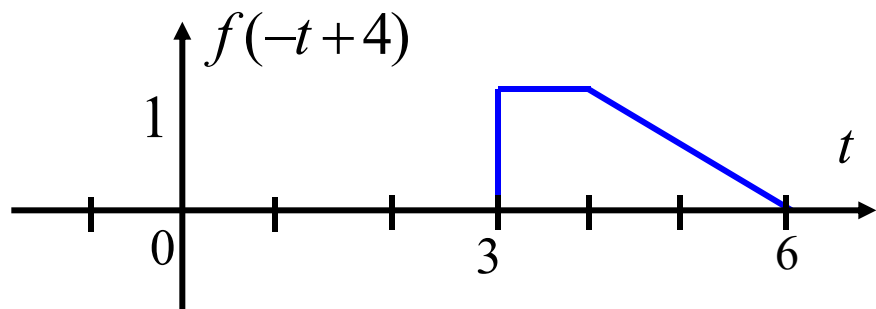
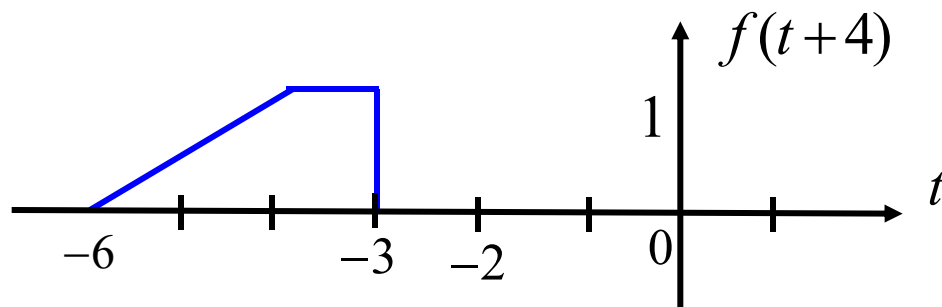
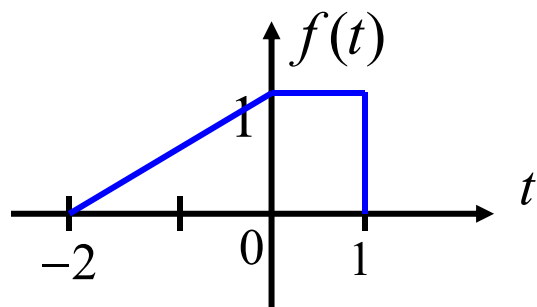




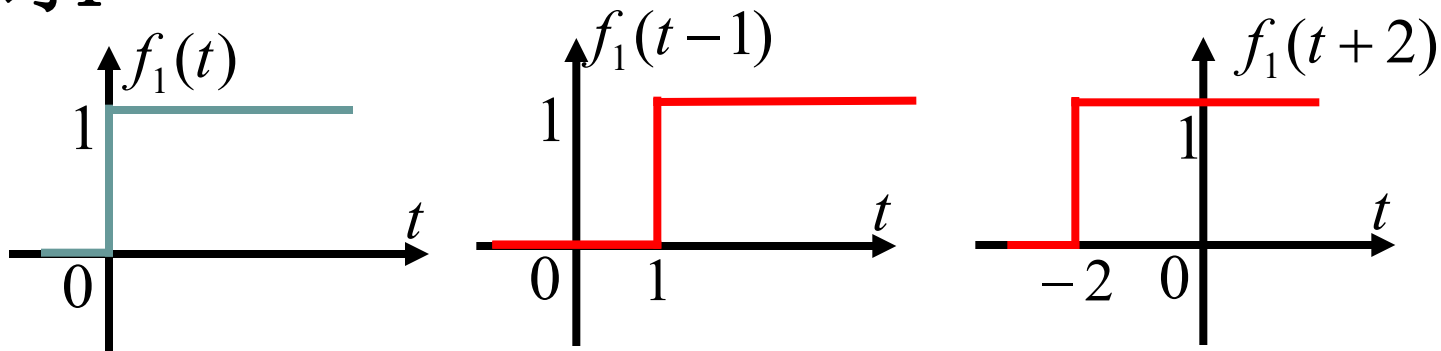
法一：



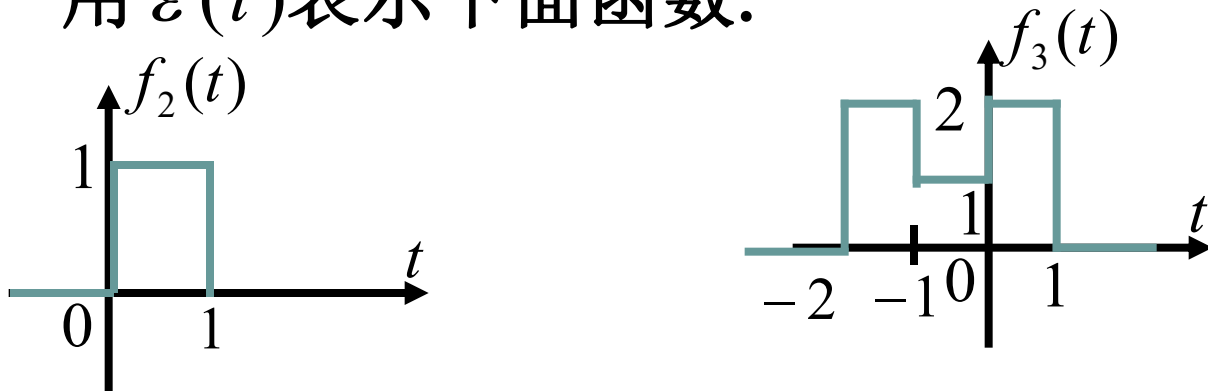
法二:



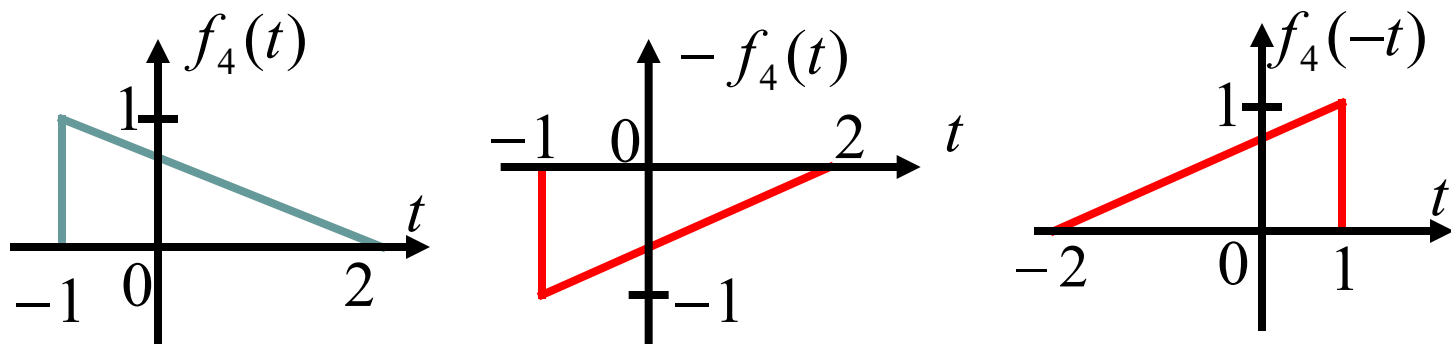
练习1



练习2 用 $\varepsilon(t)$ 表示下面函数.

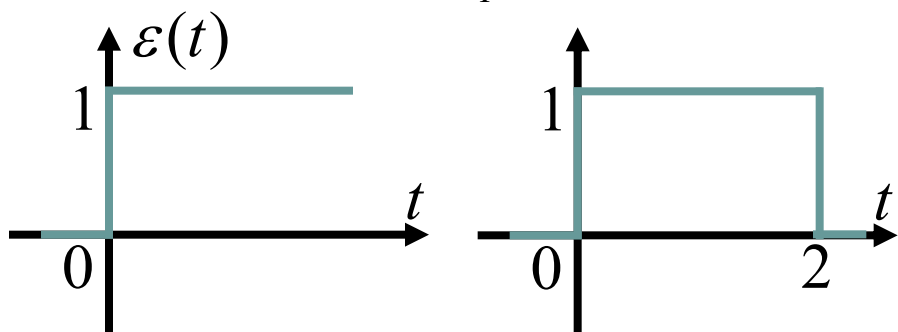


练习3

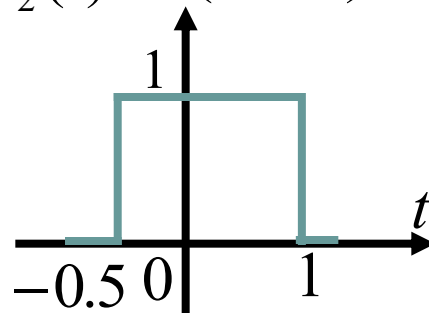


练习4

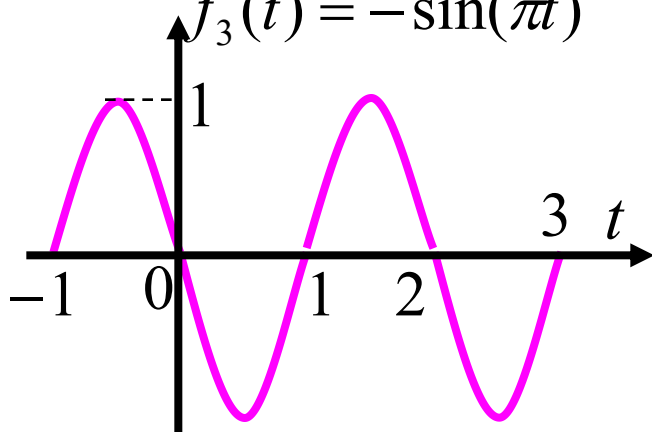
$$f_1(t) = \varepsilon(t) - \varepsilon(t-2)$$



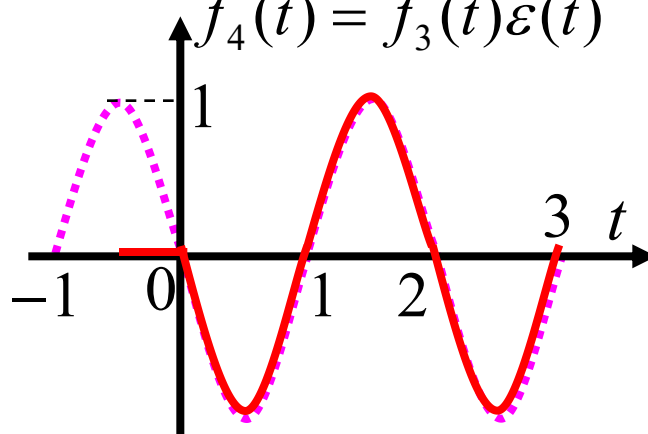
$$f_2(t) = \varepsilon(t+0.5) - \varepsilon(t-1)$$



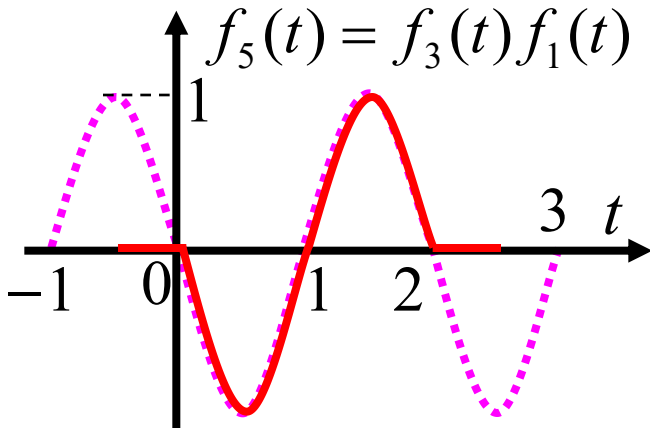
$$f_3(t) = -\sin(\pi t)$$



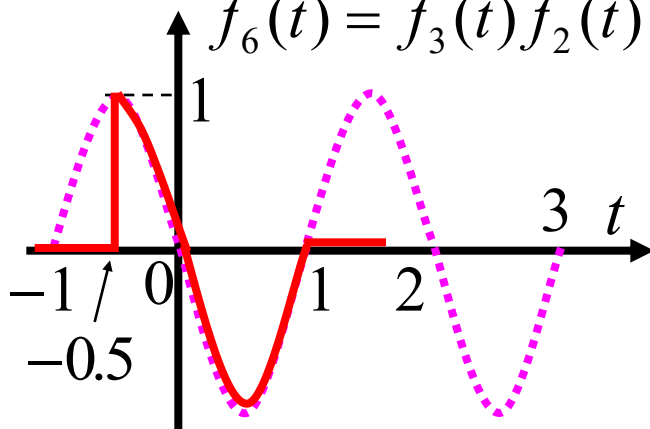
$$f_4(t) = f_3(t)\varepsilon(t)$$



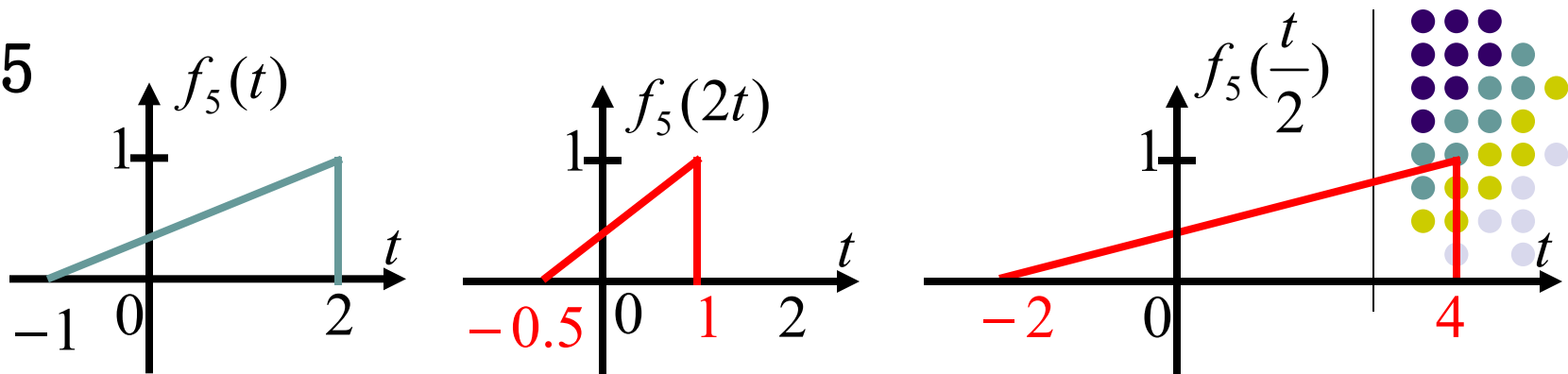
$$f_5(t) = f_3(t)f_1(t)$$



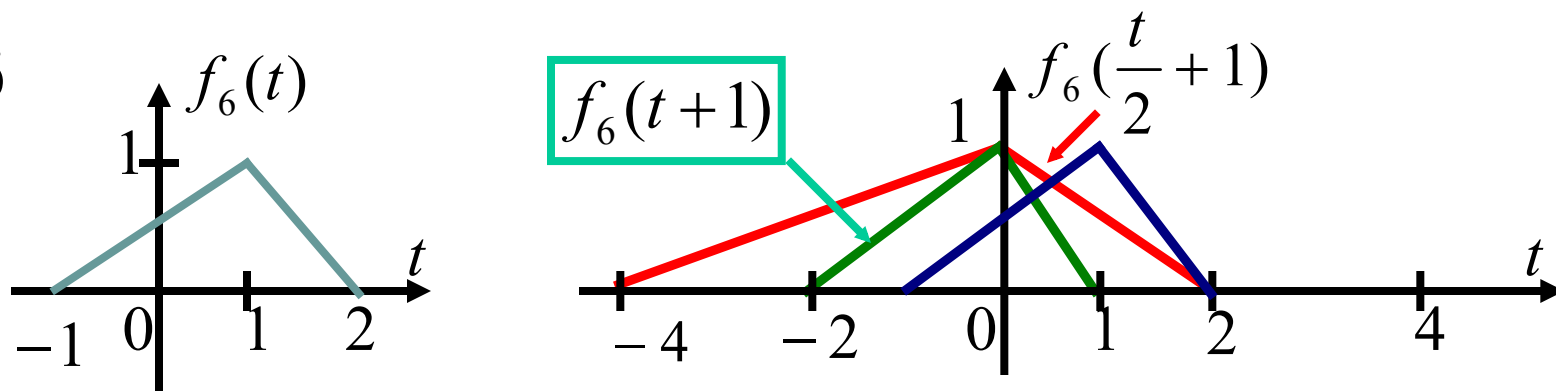
$$f_6(t) = f_3(t)f_2(t)$$



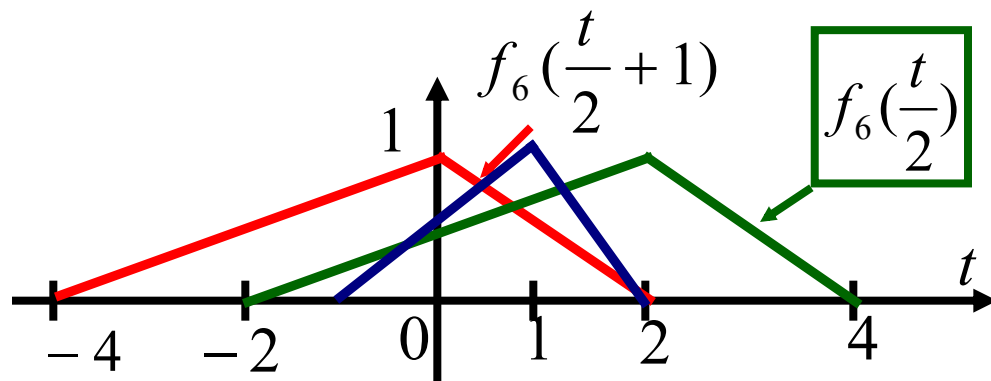
练习5

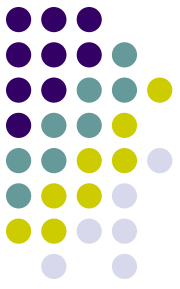


练习6



画 $f_6(\frac{t}{2} + 1)$ 波形



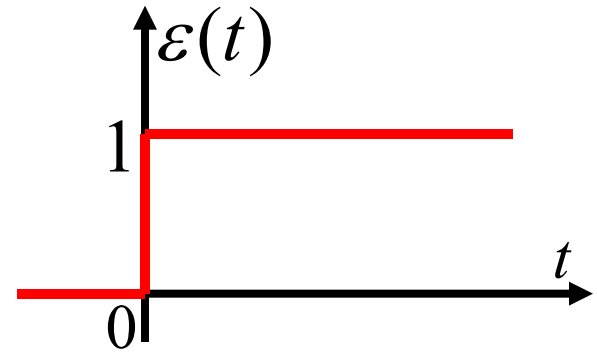


1.2 阶跃函数和冲激函数(§ 1.4)

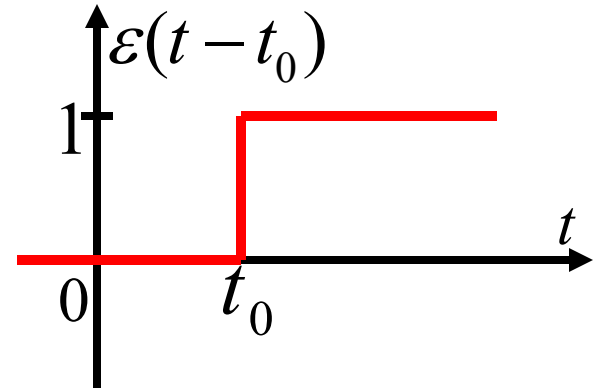
一. 阶跃函数 $\varepsilon(t)[u(t)]$ *Unit step signal*

1. 定义

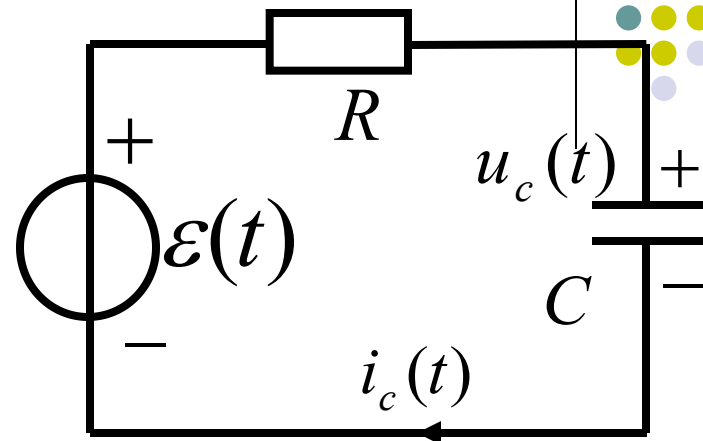
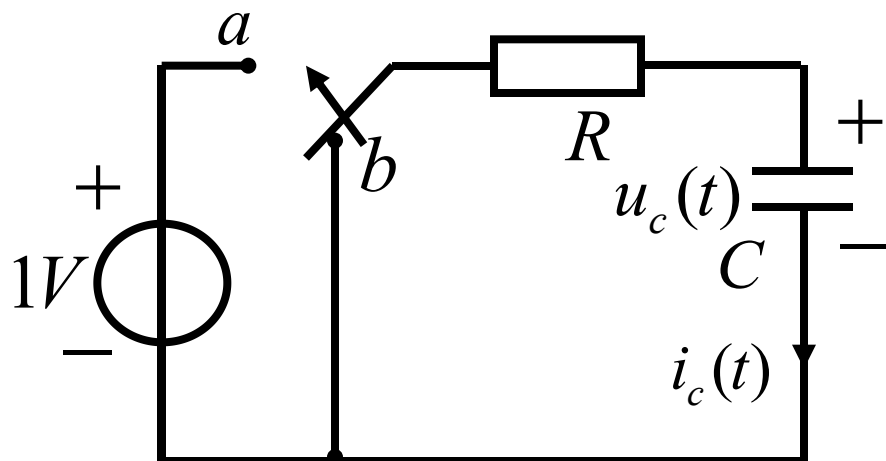
$$\varepsilon(t) = \begin{cases} 1, & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases}$$



$$\varepsilon(t - t_0) = \begin{cases} 1, & t > t_0 \\ \frac{1}{2}, & t = t_0 \\ 0, & t < t_0 \end{cases}$$



2. 实现(物理模型)



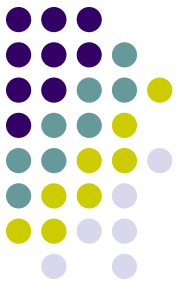
$$u_c(t) = \left[1 - e^{-\frac{t}{RC}} \right] \underline{\varepsilon(t)}$$

$$i_c(t) = \frac{1}{R} e^{-\frac{t}{RC}} \underline{\varepsilon(t)}$$

$$C = 1F, \quad R \longrightarrow 0$$

$$u_c(t) = \varepsilon(t)$$

$$i_c(t) = C \frac{d}{dt} u_c(t) = \delta(t)$$

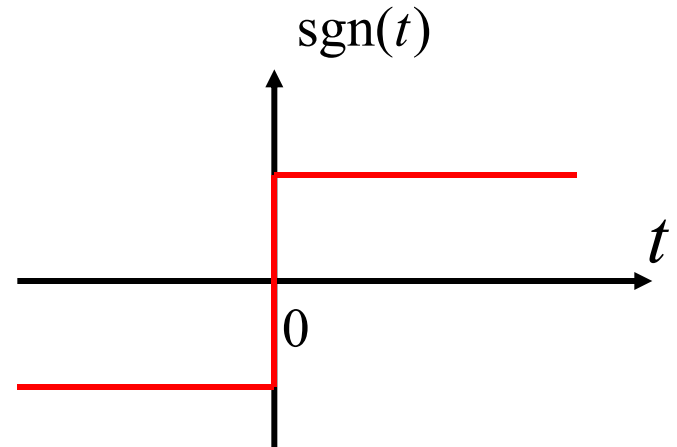


3. $\varepsilon(t)$ 应用举例

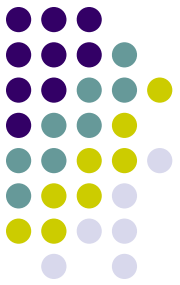
(1) 方便地表示其它函数

例 符号函数 $\text{sgn}(t)$

$$\text{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

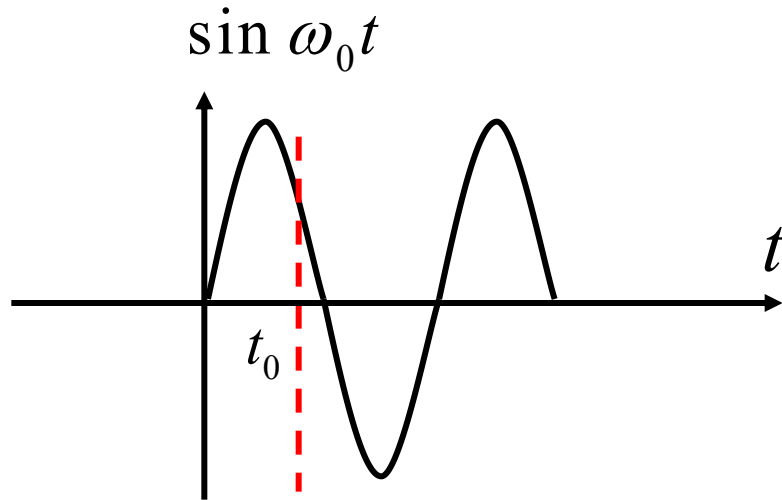


$$\text{sgn}(t) = 2\varepsilon(t) - 1$$

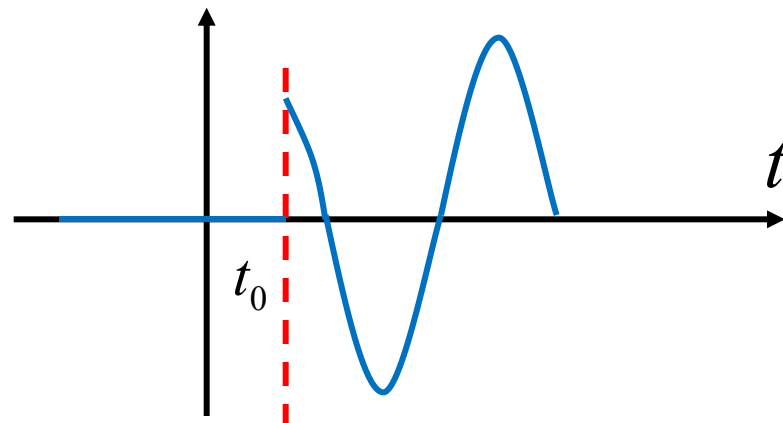


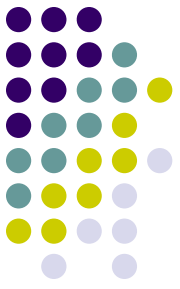
(2) 用阶跃函数表示信号的作用区间

$$\sin \omega_0 t \mathcal{E}(t - t_0)$$

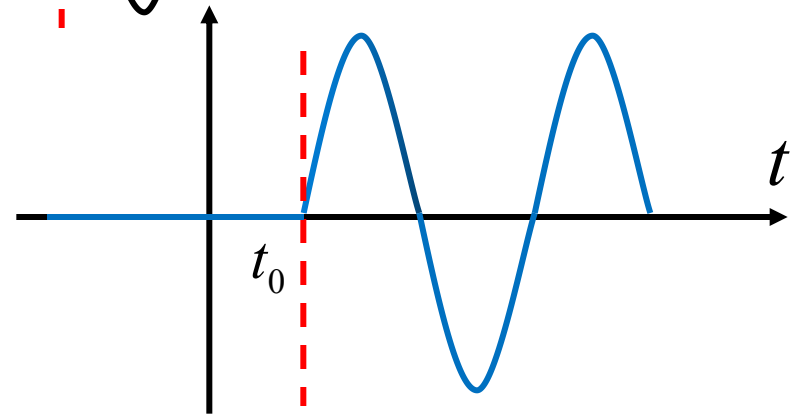
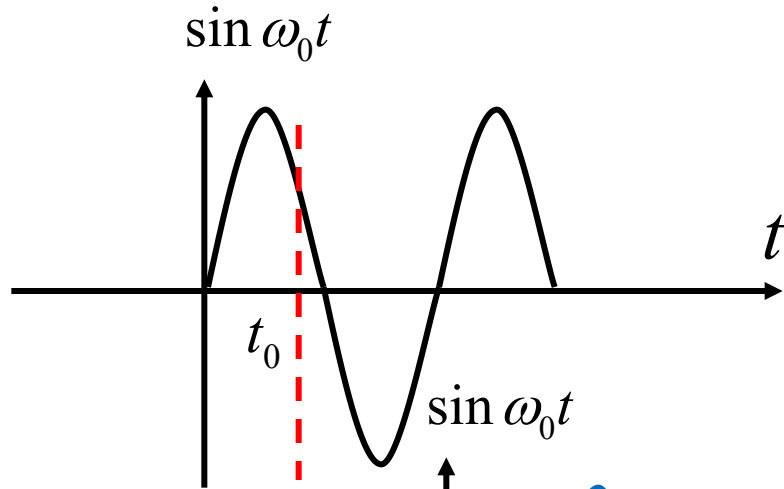
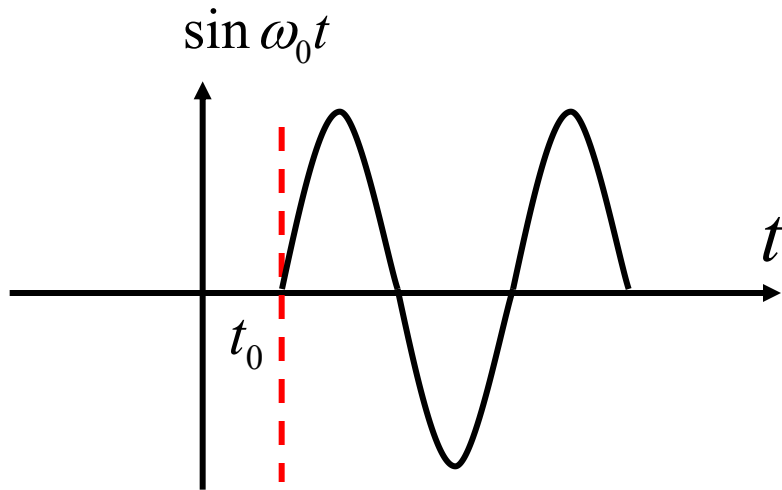


$$\sin \omega_0 t \mathcal{E}(t - t_0)$$



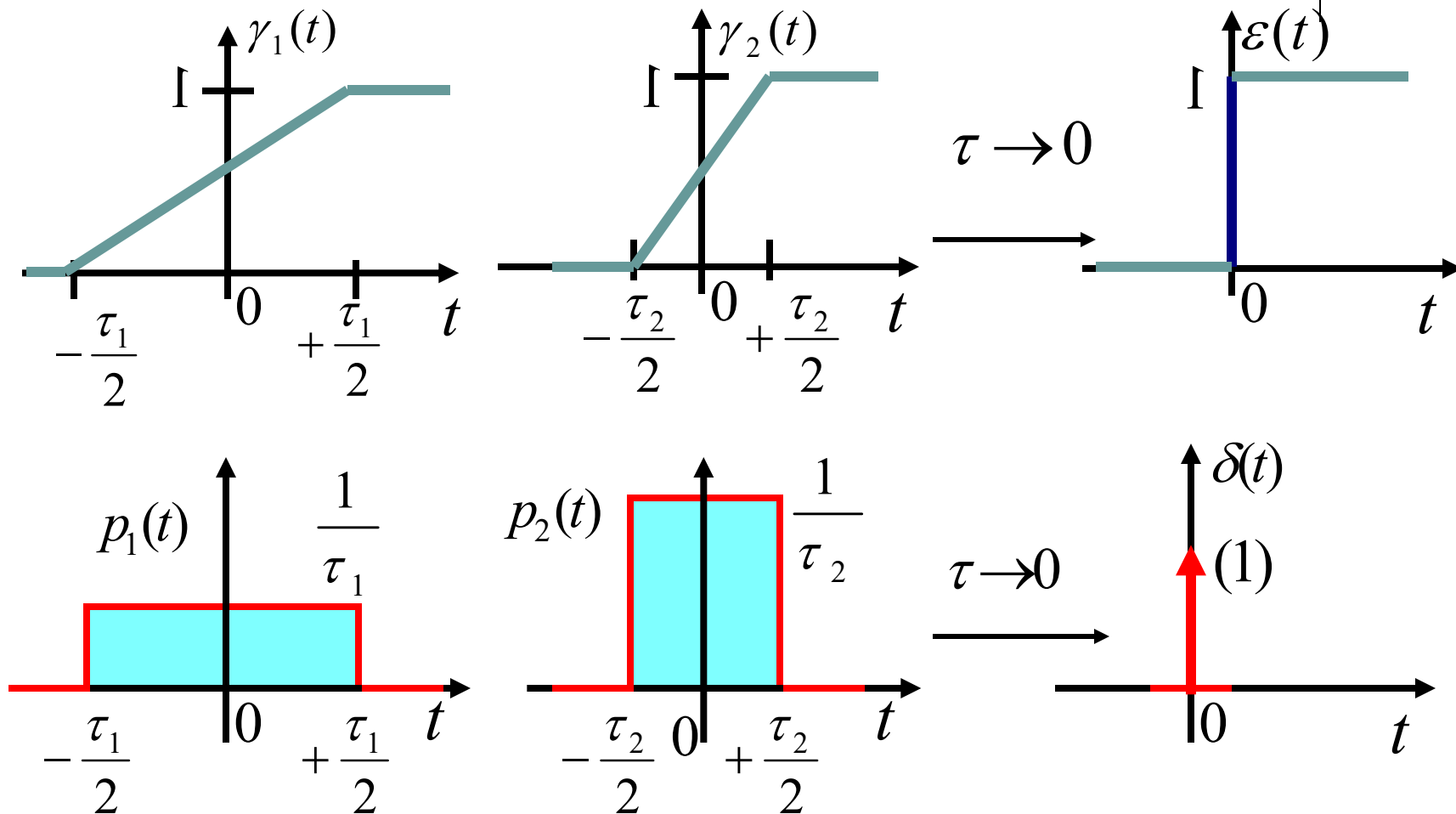


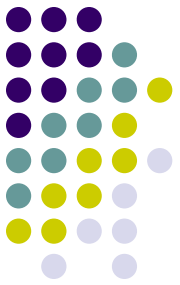
$$\sin \omega_0(t - t_0)\varepsilon(t - t_0)$$



二. 冲激函数 $\delta(t)$ *Unit Impulse Signal*.

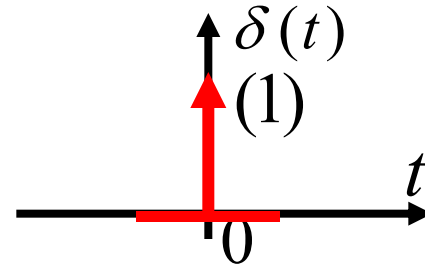
1. 定义1 (视为一般函数序列的极限)



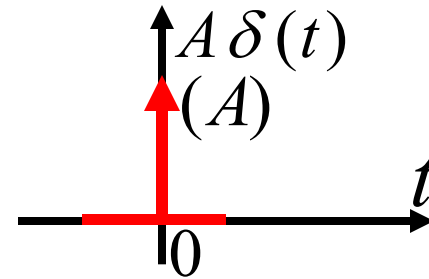


2. 定义2 狄拉克(Dirac)函数

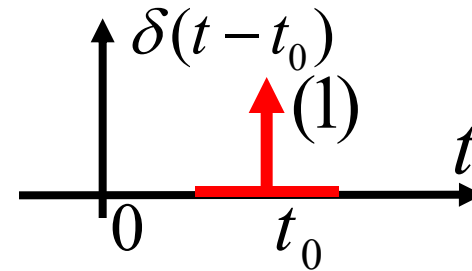
$$\delta(t) : \begin{cases} \delta(t) = 0, t \neq 0 \\ \int_{-\infty}^{+\infty} \delta(t) dt = 1 \end{cases}$$



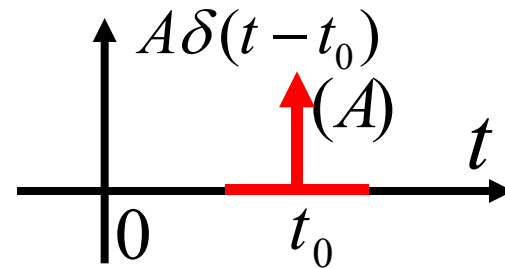
$$A\delta(t) : \begin{cases} A\delta(t) = 0, t \neq 0 \\ \int_{-\infty}^{+\infty} A\delta(t) dt = A \end{cases}$$



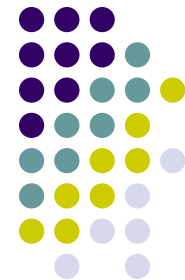
$$\delta(t-t_0) : \begin{cases} \delta(t-t_0) = 0, t \neq t_0 \\ \int_{-\infty}^{+\infty} \delta(t-t_0) dt = 1 \end{cases}$$



$$A\delta(t-t_0) : \begin{cases} A\delta(t-t_0) = 0, t \neq t_0 \\ \int_{-\infty}^{+\infty} A\delta(t-t_0) dt = A \end{cases}$$



三. $\delta(t)$ 函数的性质



$$1. \quad \delta(t) = \frac{d}{dt} \varepsilon(t)$$

$$\varepsilon(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

$$\delta(t - t_0) = \frac{d}{dt} \varepsilon(t - t_0)$$

$$\varepsilon(t - t_0) = \int_{-\infty}^t \delta(\tau - t_0) d\tau$$

$$2. \quad \delta(t) = \delta(-t)$$

$$3. \quad x(t)\delta(t) = x(0)\delta(t)$$

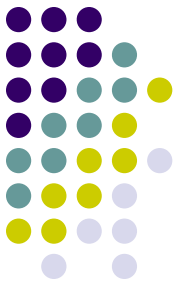
例1.2-1 计算(P19例1.4-1)

$$t\delta(t) = 0$$

$$e^{-\alpha t} \delta(t) = \delta(t)$$

$$\delta(t) \cos t = \delta(t)$$

$$\delta(t) \sin 3t = 0$$



$$4. \quad x(t) \delta(t-t_0) = x(t_0) \delta(t-t_0)$$

例 1.2-2 计算

$$\cos t \delta\left(t - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \delta\left(t - \frac{\pi}{4}\right)$$
$$\delta\left(t + \frac{\pi}{4}\right) \sin t = -\frac{\sqrt{2}}{2} \delta\left(t + \frac{\pi}{4}\right)$$

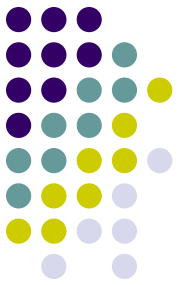
$$5. \quad \int_{-\infty}^{+\infty} x(t) \delta(t) dt = x(0) \quad \int_{-\infty}^{+\infty} x(t) \delta(t-t_0) dt = x(t_0)$$

例 1.2-3 计算

$$\int_{-\infty}^{+\infty} 2t \delta(t-1) dt = 2$$
$$\int_{-\infty}^{+\infty} \delta(t+3) e^{-t} dt = e^3$$

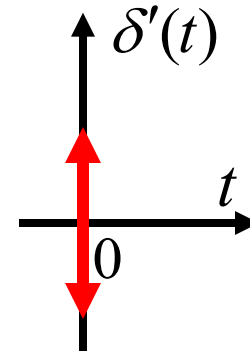
练习 计算

$$\int_{-\infty}^{+\infty} \delta(1-t)(t^3 + 4) dt = 5$$



6. $\delta(t)$ 的导数

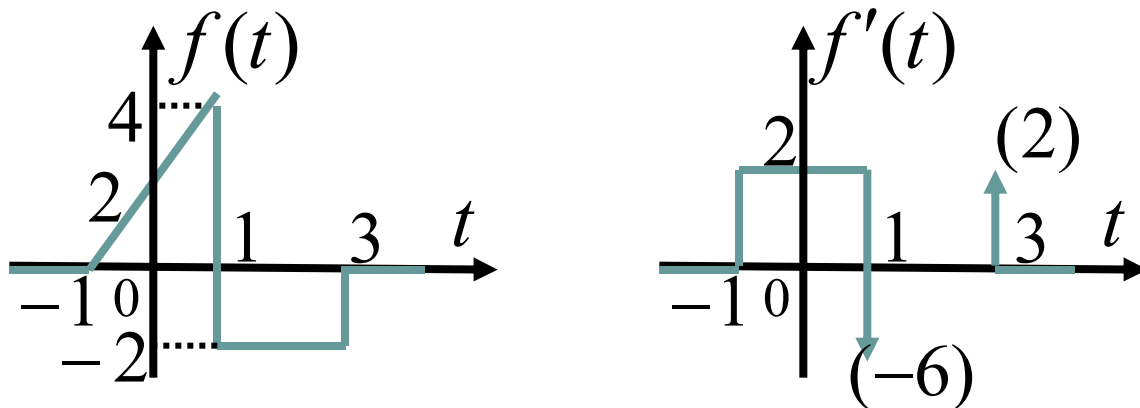
$$\delta'(t) = \frac{d}{dt} \delta(t) \quad \int_{-\infty}^{+\infty} \delta'(t) dt = 0$$



$$\int_{-\infty}^{+\infty} \delta'(t) x(t) dt = -x'(0)$$

例 1.2-4 计算 $\frac{d}{dt} [\cos(\omega_0 t) \cdot \varepsilon(t)] = \delta(t) - \omega_0 \sin(\omega_0 t) \varepsilon(t)$

例1.2-5化出 $f'(t)$ 的波形图(P20例1.4-2)



$$f(t) = (2t + 2)[\varepsilon(t + 1) - \varepsilon(t - 1)] - 2[\varepsilon(t - 1) - \varepsilon(t - 3)]$$

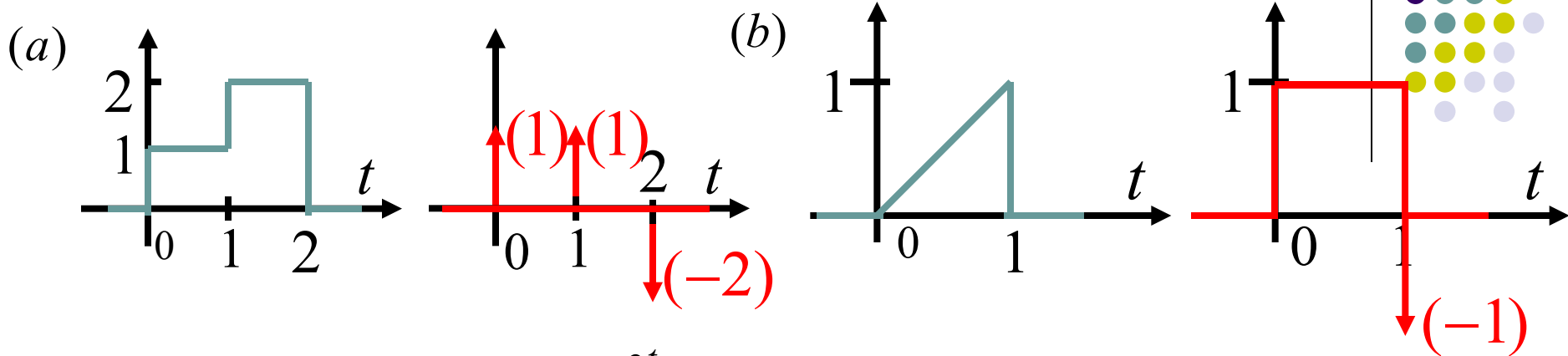
7.尺度变换 $\delta(at) = \frac{1}{|a|} \delta(t)$

8.卷积 (1). $f_1(t) * f_2(t) = \int_{-\infty}^{+\infty} f_1(\tau) f_2(t - \tau) d\tau$

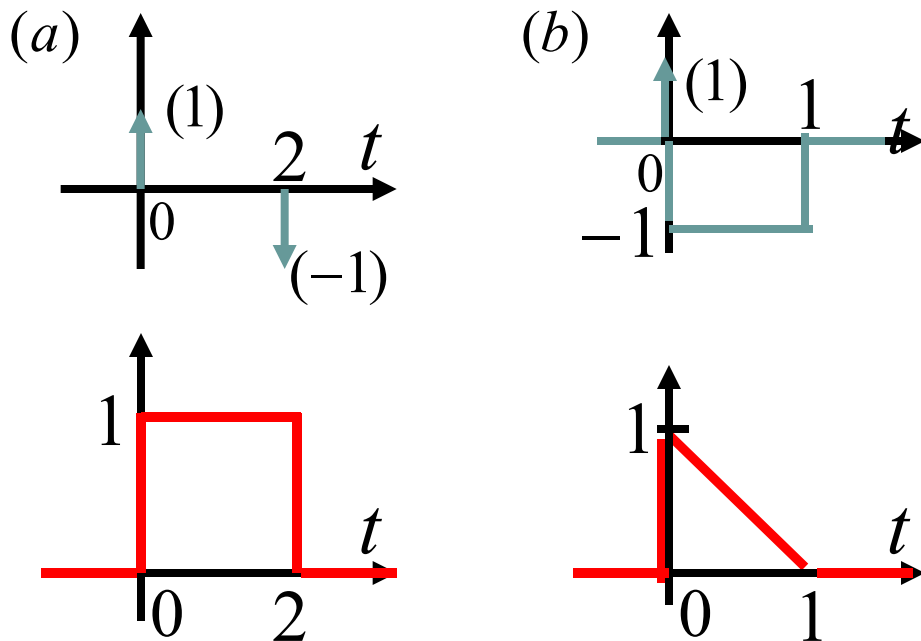
(2). $x(t) * \delta(t) = \delta(t) * x(t) = x(t)$

(3). $x(t) * \delta(t - t_0) = x(t - t_0)$

练习：求各函数的一阶导数



练习：求各函数的积分 $\int_{-\infty}^t f(x)dx$





1.3 系统的特性、分析和描述

一. 系统的分类 (§ 1.6) *continuous time system*

1. (时间) 连续系统&离散系统&混合系统 *discrete ~ ~*



2. 因果系统&非因果系统 *causal system*

因果系统(物理可实现系统):

若当 $t < 0$ 时, $f(t) = 0$, 则当 $t < 0$ 时, 一定有 $y(t) = 0$.

3. 稳定系统&非稳定系统

稳定系统: 若 $|f(t)| < \infty$ 则 $|y_{zs}(t)| < \infty$

4. 非时变系统&时变系统 *time invariant system*

非时变系统: 若 $f(t) \rightarrow y(t)$ 则 $f(t - t_0) \rightarrow y(t - t_0)$

5. 线性系统 & 非线性系统

linear system



(1).同时满足齐次性和可加性的系统,即若:

$$f_1(t) \rightarrow y_1(t), f_2(t) \rightarrow y_2(t)$$

则 $a_1 f_1(t) + a_2 f_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$

(2).可分解性: $y(t) = y_{zi}(t) + y_{zs}(t)$ $y_x(t) \leftrightarrow y_{zi}(t)$

$y_{zi}(t), y_{zs}(t)$ 均满足线性.

$$y_f(t) \leftrightarrow y_{zs}(t)$$

6. 线性非时变系统 (LTI) 性质推论

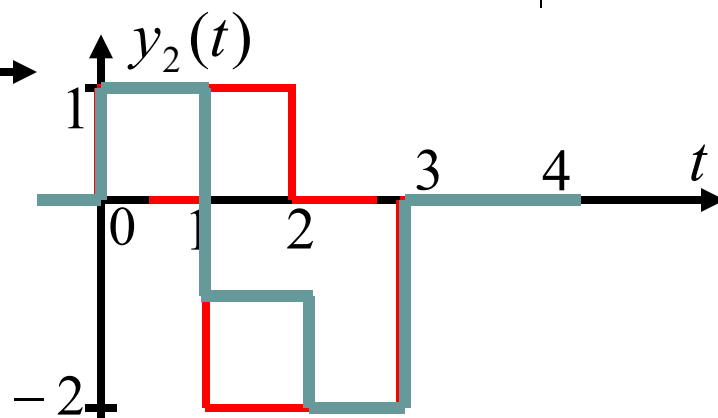
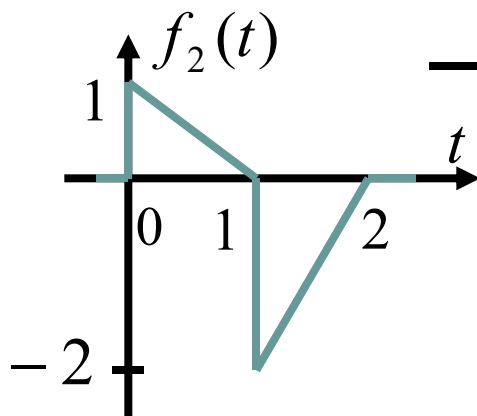
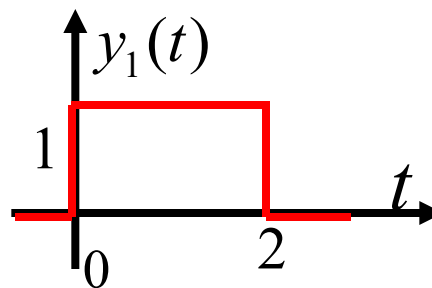
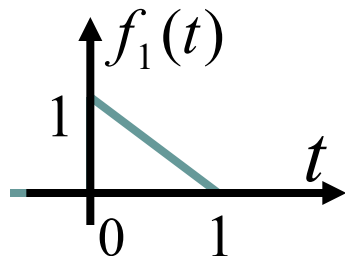
若 $f(t) \rightarrow y(t)$ *linear time invariant system*
则

$$af(t) \rightarrow ay(t)$$

$$f'(t) \rightarrow y'(t)$$

$$\int_{-\infty}^t f(x)dx \rightarrow \int_{-\infty}^t y(x)dx$$

例1.3-1



*linear
time invariant system*

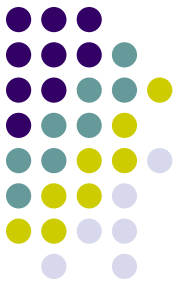
例1.3-2 设LTI系统初始状态 $x(0)$ 不为零,输入 $f(t)$ 时,全响应

$$y(t) = y_{zi}(t) + y_{zs}(t)$$

若初始状态不变,输入为 $2f(t)$,全响应

$$y(t) = y_{zi}(t) + 2y_{zs}(t)$$



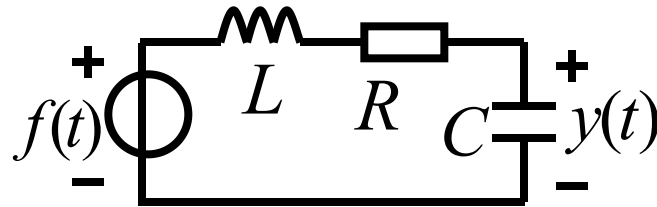


二. 系统的数学模型—方程（组）描述

（§ 1.5 系统的描述）

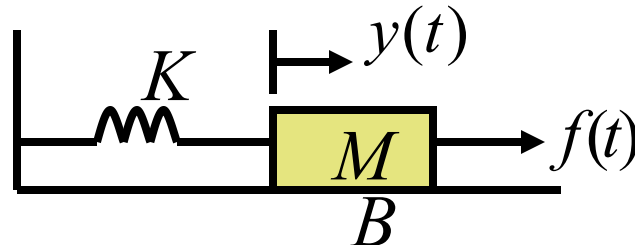
连续系统 线性常微分方程 *Differential Equation*

例1.3-3

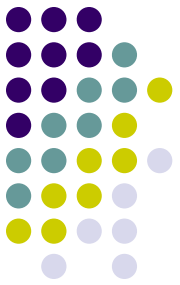


$$LCy''(t) + RCy'(t) + y(t) = f(t)$$

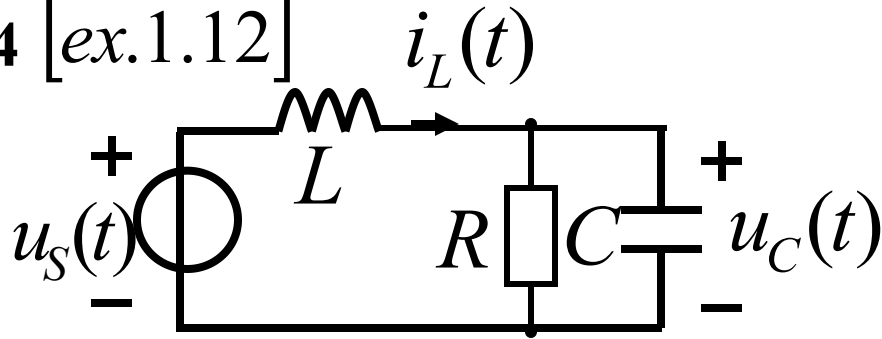
相似系统 具有相同的数学模型的系统.



$$My''(t) + By'(t) + Ky(t) = f(t)$$



例1.3-4 [ex.1.12]



输入为 $u_s(t)$ 分别以 $i_L(t)$ 和 $u_C(t)$ 为输出(响应), 建立方程

$$LCu_C''(t) + \frac{L}{R}u_C'(t) + u_C(t) = u_s(t)$$

$$u_C''(t) + \frac{1}{RC}u_C'(t) + \frac{1}{LC}u_C(t) = \frac{1}{LC}u_s(t)$$

$$LCi_L''(t) + \frac{L}{R}i_L'(t) + i_L(t) = \frac{1}{R}u_s(t) + Cu_s'(t)$$

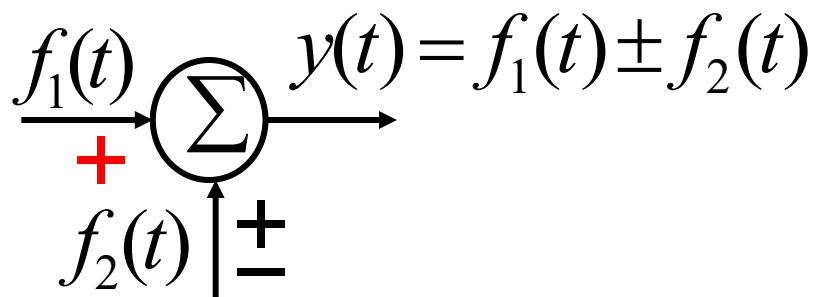
$$i_L''(t) + \frac{1}{RC}i_L'(t) + \frac{1}{LC}i_L(t) = \frac{1}{RLC}u_s(t) + \frac{1}{L}Cu_s'(t)$$

三. 系统的框图表示

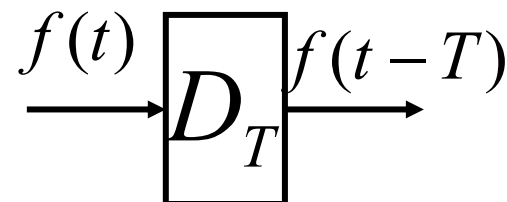


1. 基本部件

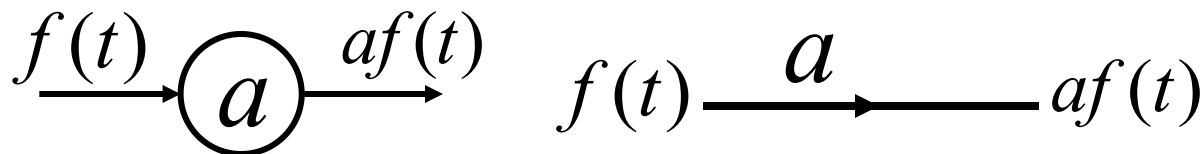
求和器



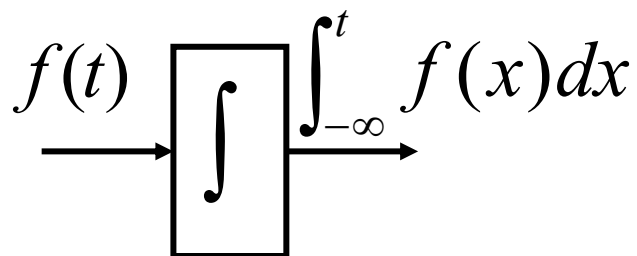
延迟器



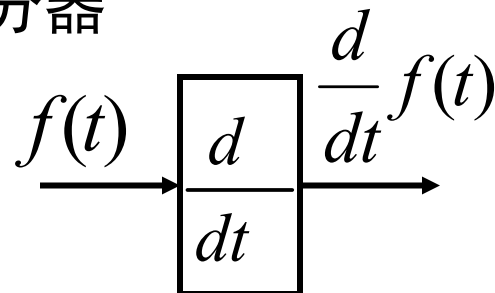
数乘器

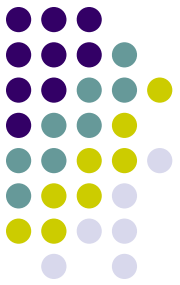


积分器



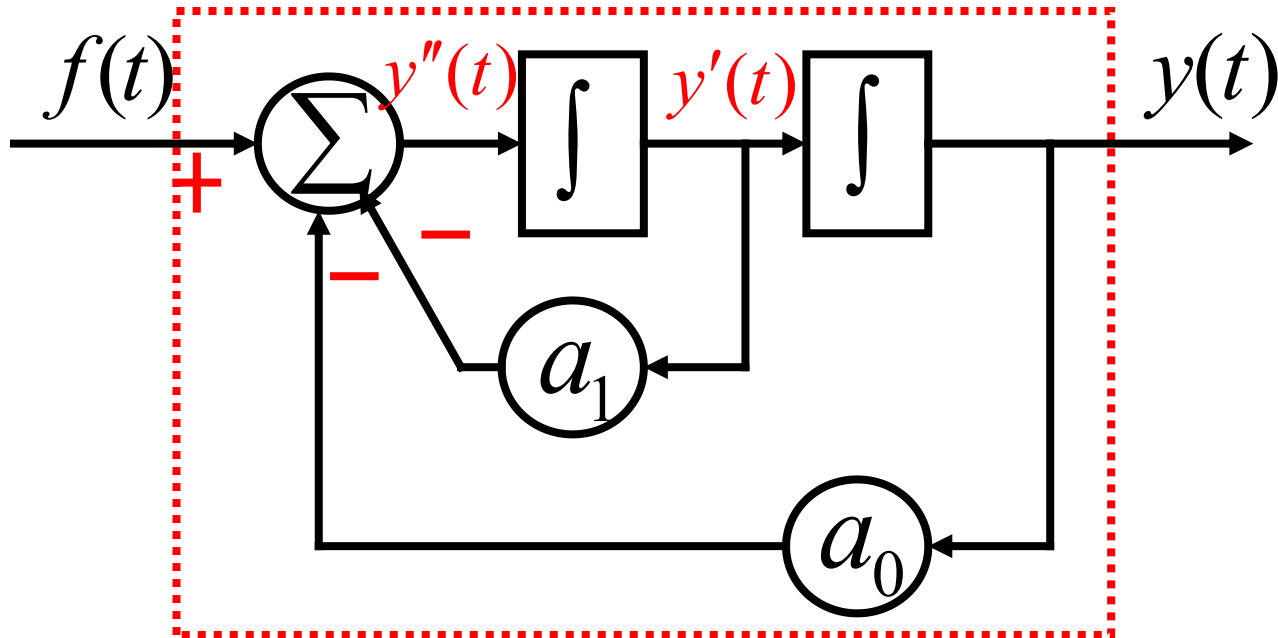
微分器





2. 由框图写微分方程

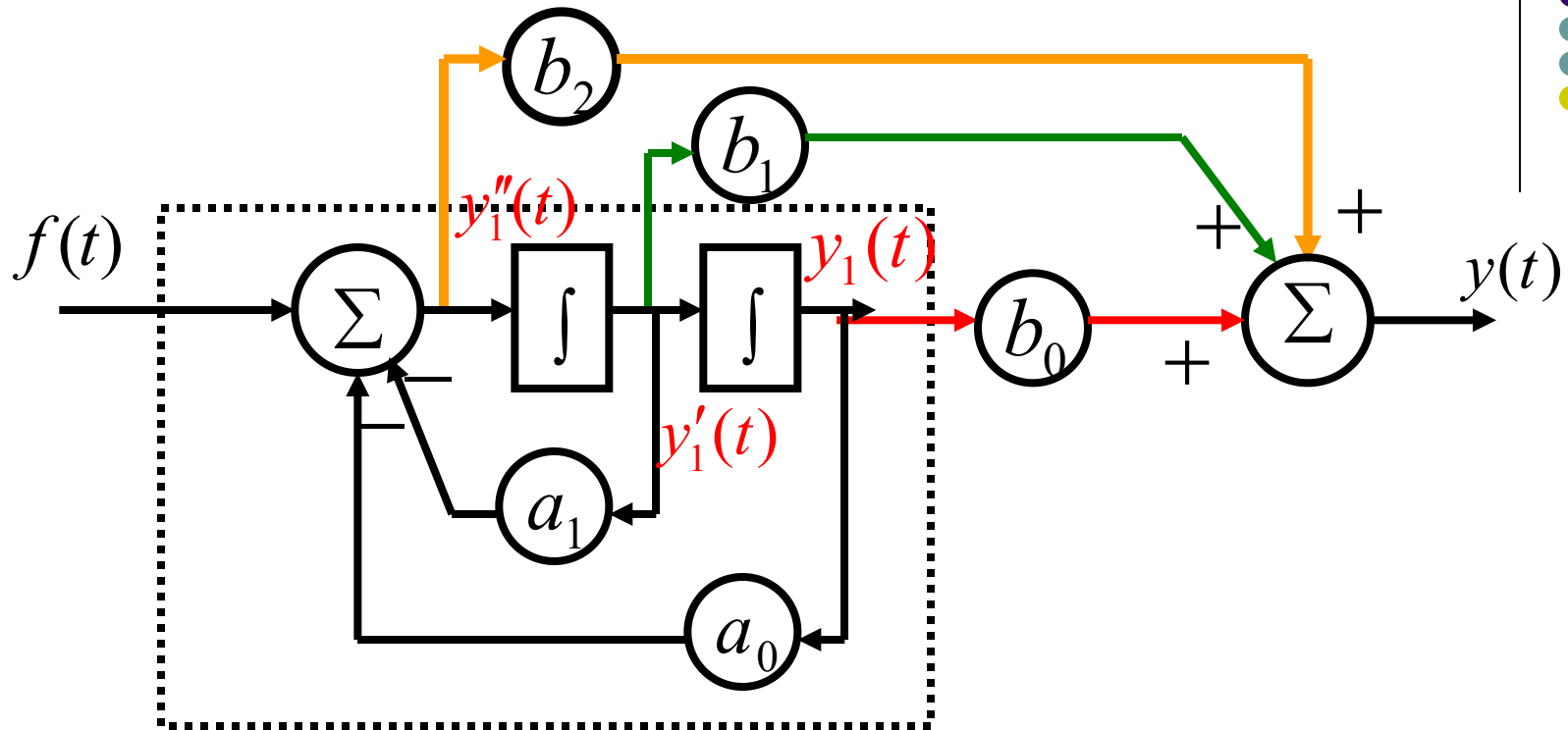
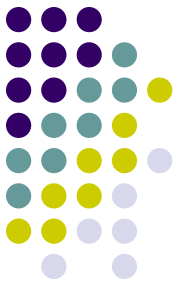
例1.3-5 写出描述系统的微分方程. (P25例1.5-1)



$$y''(t) = -a_1 y'(t) - a_0 y(t) + f(t) \quad (1-1)$$

$$y''(t) + a_1 y'(t) + a_0 y(t) = f(t)$$

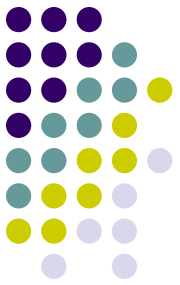
例1.3-6 写出描述系统的微分方程.(P25例1.5-2)



$$y_1''(t) = -a_1 y_1'(t) - a_0 y_1(t) + f(t) \quad (1-1)$$

$$y_1''(t) + a_1 y_1'(t) + a_0 y_1(t) = f(t) \quad (1)$$

$$y''(t) + a_1 y'(t) + a_0 y(t) = b_2 f''(t) + b_1 f'(t) + b_0 f(t)$$



第二章 连续时间系统的时域分析

要点：

1. 微分方程的解
2. 单位冲激响应 $h(t)$,
单位阶跃响应 $g(t)$
3. 卷积运算(定义, 性质)
4. 连续时间系统的时域分析

2.1 卷积积分 (§ 2.3 § 2.4)



一. 卷积(积分)定义 *Convolution*

定义: 对于函数 $f_1(t), f_2(t)$, $-\infty < t < \infty$

定义卷积: $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t - \tau) d\tau$

例2.1-1 已知: $f_1(t) = \varepsilon(t), f_2(t) = e^{-t} \varepsilon(t)$

$$f_3(t) = e^{-2t} \varepsilon(t)$$

计算: $y_1(t) = f_1(t) * f_2(t) = (1 - e^{-t}) \underline{\varepsilon(t)}$

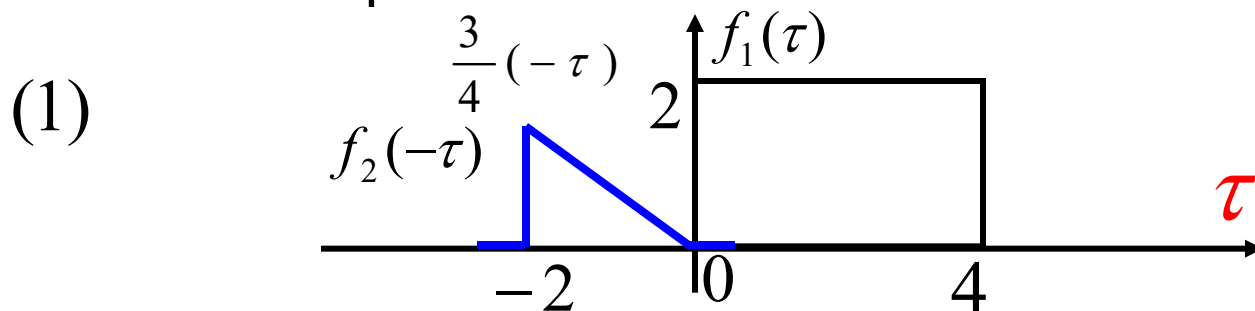
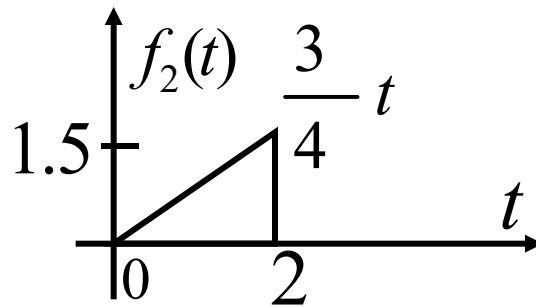
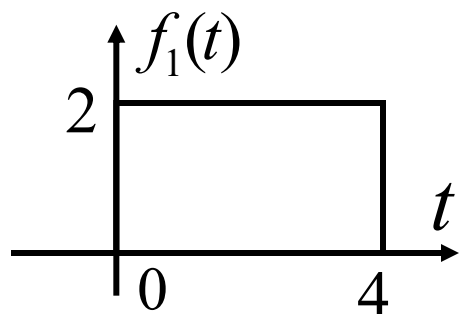
$$y_2(t) = f_2(t) * f_1(t) = (1 - e^{-t}) \underline{\varepsilon(t)}$$

$$y_3(t) = f_2(t) * f_3(t) = (e^{-t} - e^{-2t}) \underline{\varepsilon(t)}$$

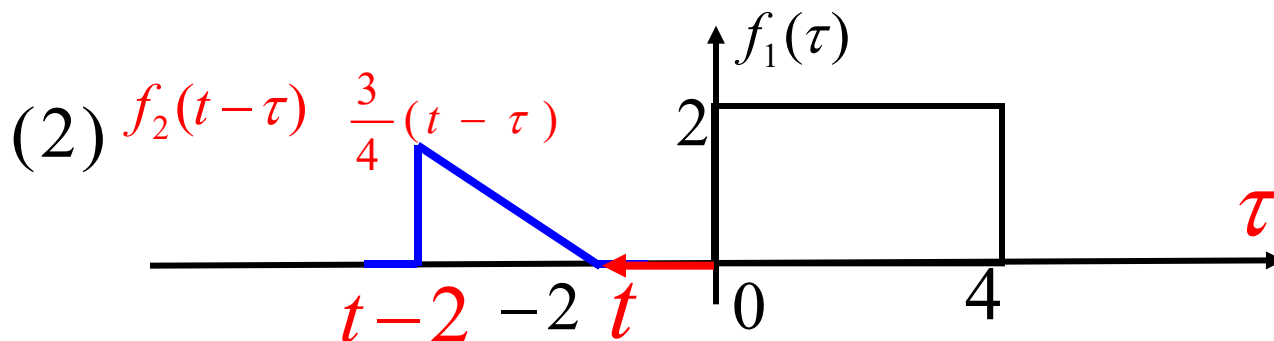
二. 卷积的图解

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$$

例2.1-2 (p61图2.3-3,图2.3-4) 计算 $y_1(t) = f_1(t) * f_2(t)$

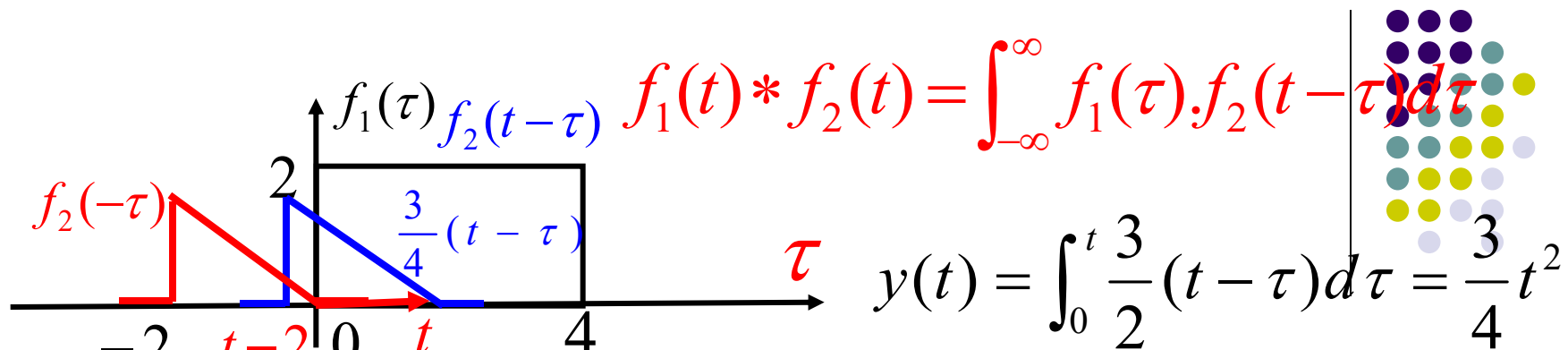


$t=0$



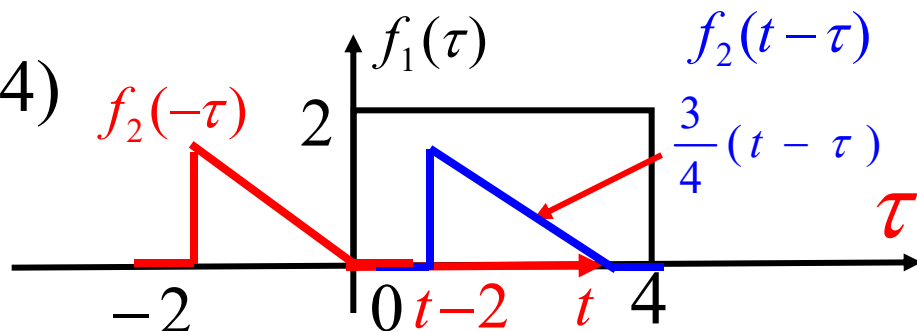
$t < 0$

(3)



$$2 > t \geq 0$$

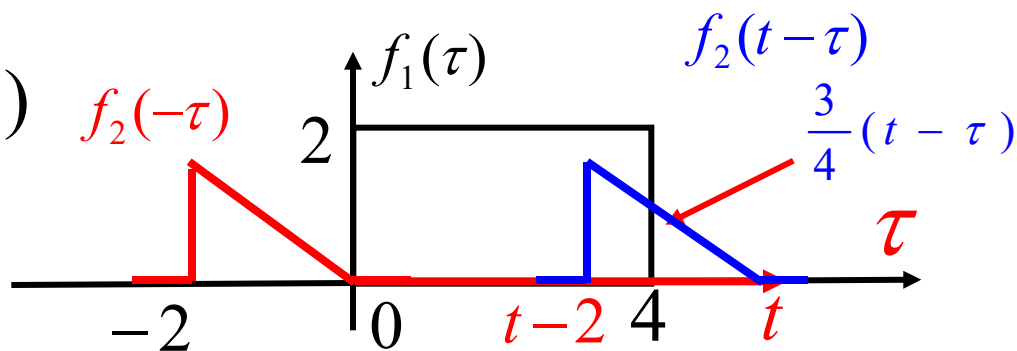
(4)



$$y(t) = \int_{t-2}^t \frac{3}{2}(t-\tau) d\tau = 3$$

$$4 > t \geq 2$$

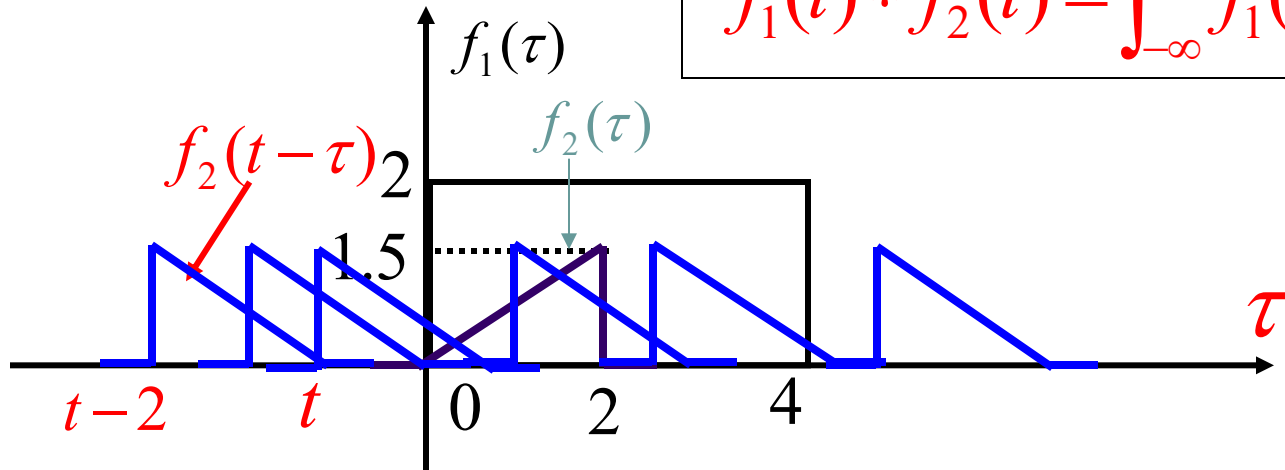
(5)



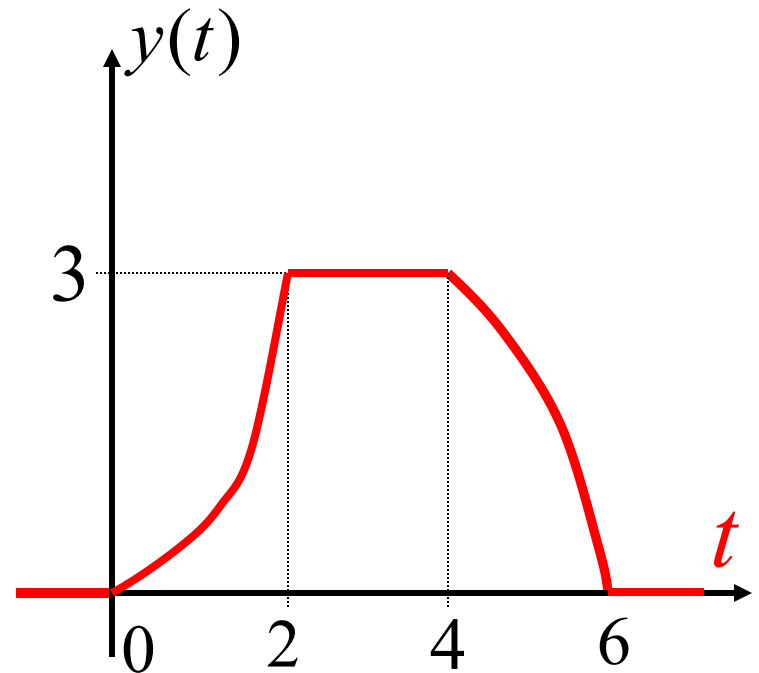
$$6 > t \geq 4$$

$$y(t) = \int_{t-2}^4 \frac{3}{2}(t-\tau) d\tau = -\frac{3}{4}t^2 + 6t - 9$$

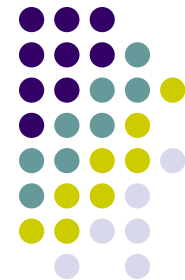
$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$$



$$y(t) = \begin{cases} 0, & t < 0 \\ \frac{3}{4}t^2, & 2 > t \geq 0 \\ 3, & 4 > t \geq 2 \\ -\frac{3}{4}t^2 + 6t - 9, & 6 > t \geq 4 \\ 0, & t \geq 6 \end{cases}$$



三. 卷积的运算性质



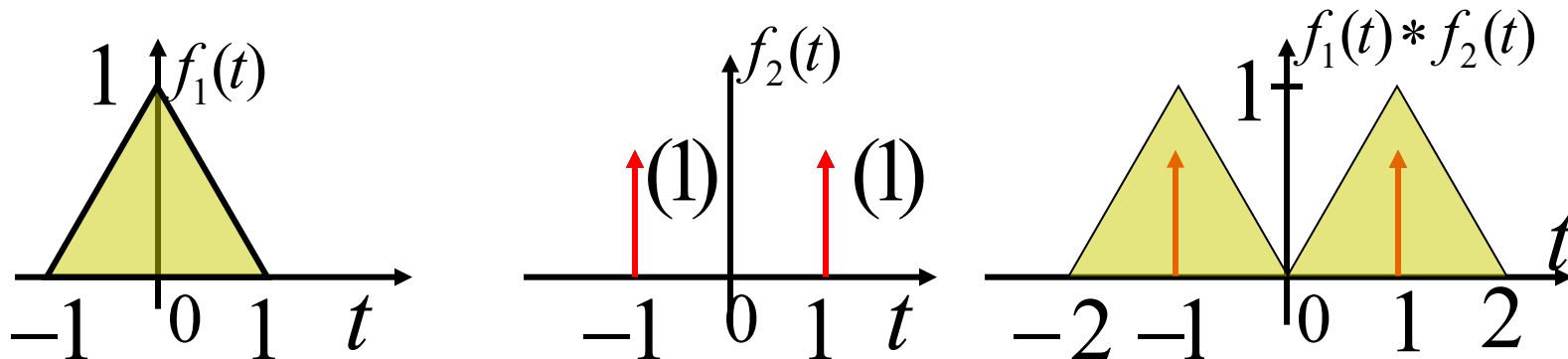
1. 交换率: $f_1(t) * f_2(t) = f_2(t) * f_1(t)$

2. 分配率: $f_1(t) * [f_2(t) + f_3(t)] = f_1(t) * f_2(t) + f_1(t) * f_3(t)$

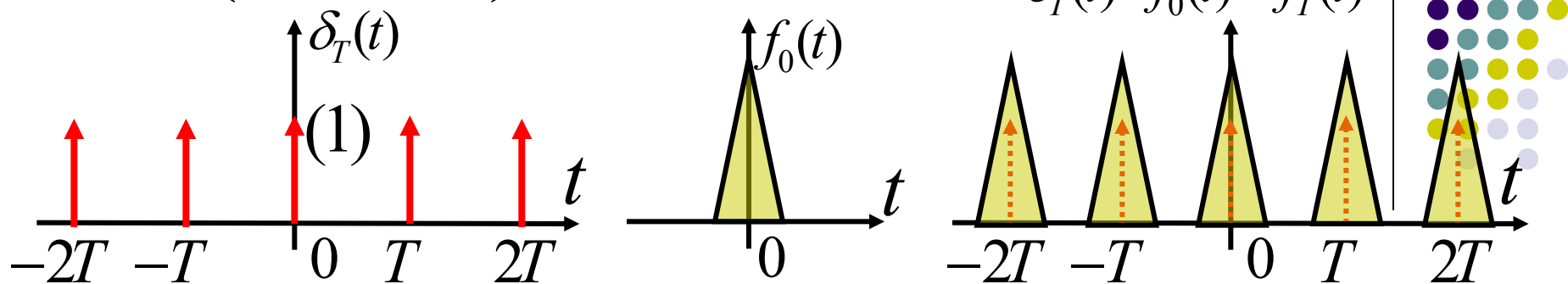
3. 结合率: $f_1(t) * [f_2(t) * f_3(t)] = [f_1(t) * f_2(t)] * f_3(t)$

4. 冲激函数
卷 积: $f(t) * \delta(t) = \delta(t) * f(t) = f(t)$
 $f(t) * \delta(t - t_0) = f(t - t_0)$

例2.1-3



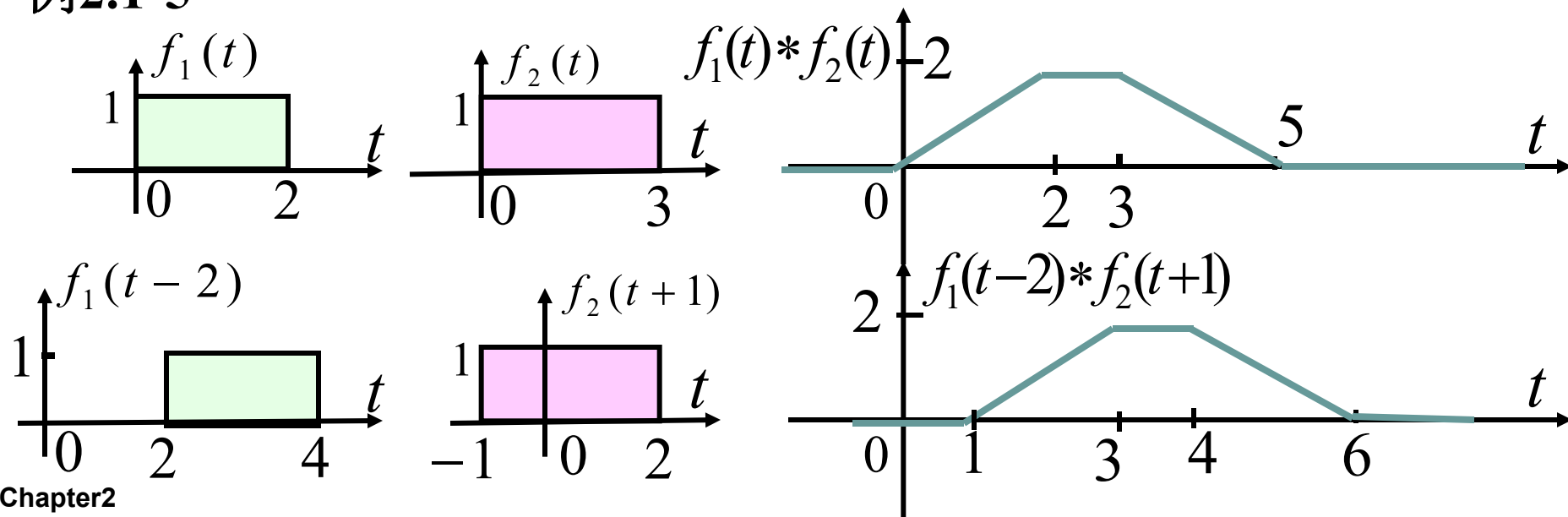
例 2.1-4(P71例2.4-3)



推论： 若 $f_1(t) * f_2(t) = y(t)$

则 $f_1(t-t_1) * f_2(t-t_2) = y(t-t_1-t_2)$

例2.1-5

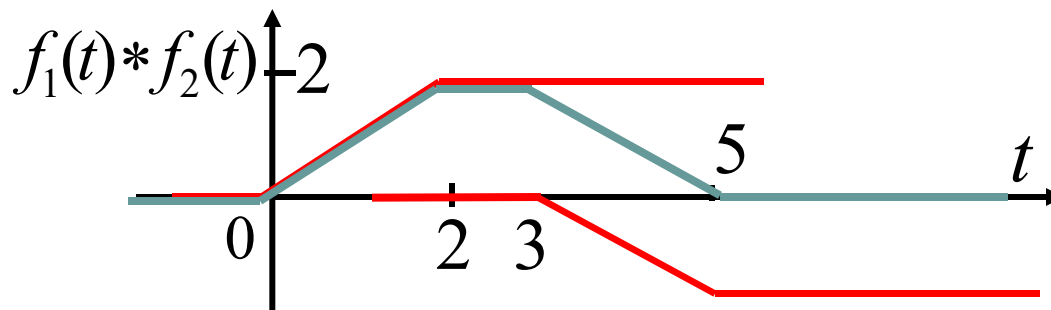
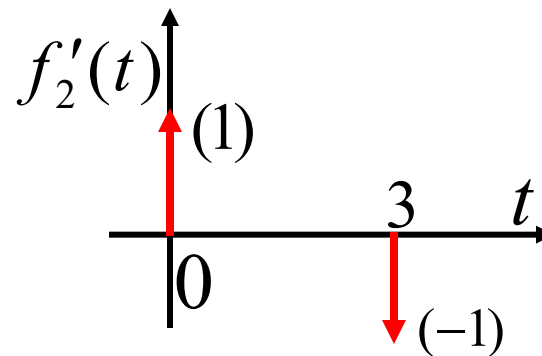
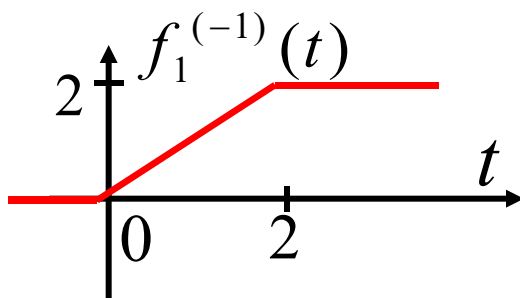
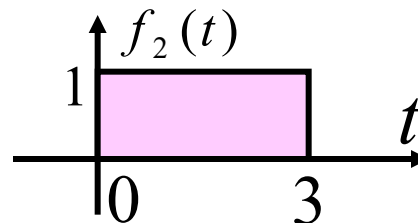
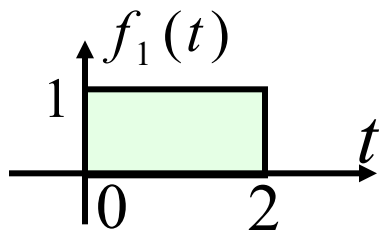


5.微分积分性质: 若 $y(t) = f_1(t) * f_2(t)$

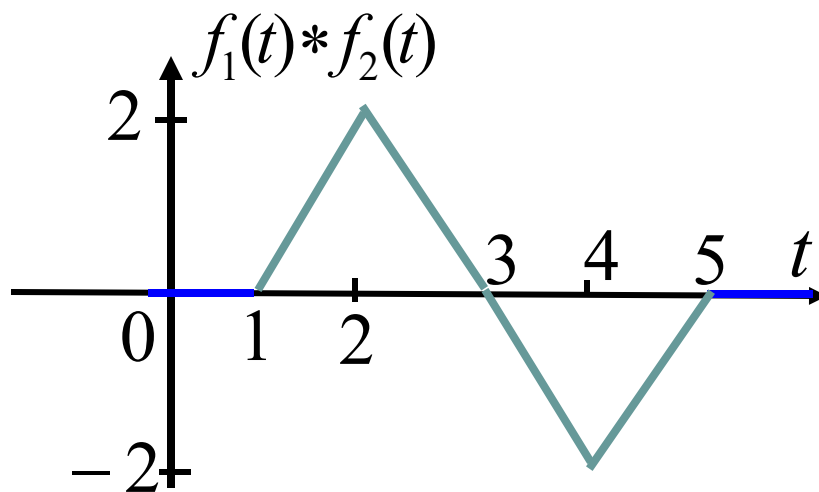
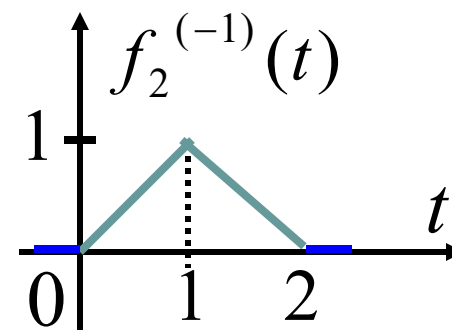
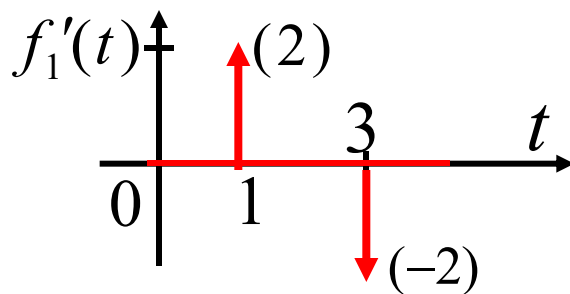
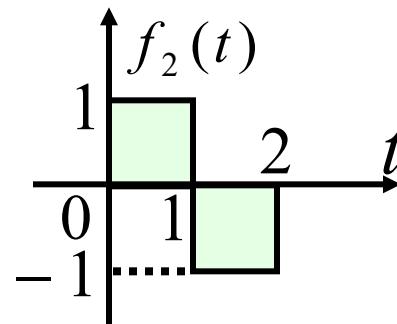
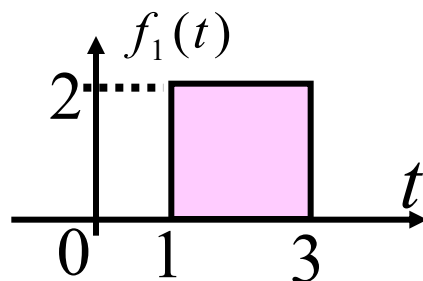
注

$$\text{则 } y(t) = f_1^{(-1)}(t) * f_2'(t) = f_1'(t) * f_2^{(-1)}(t)$$

例2.1-6



例2.1-7(P73例2.4-4)

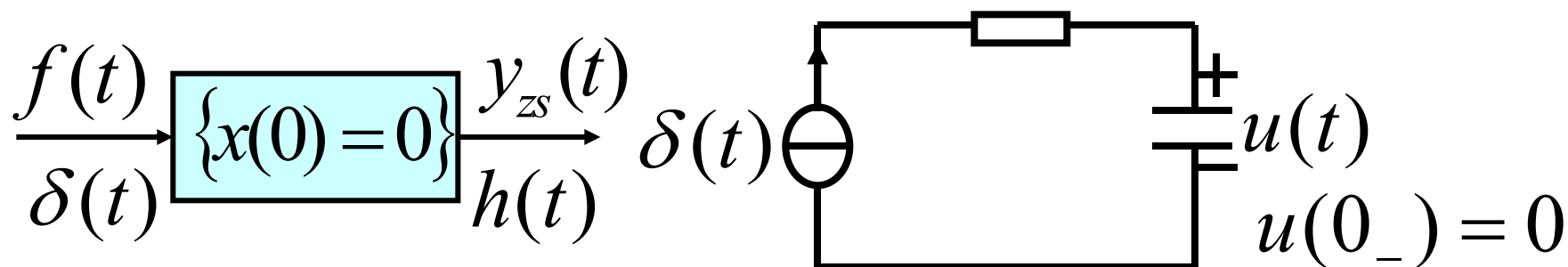


2.2 冲激响应和阶跃响应 (§ 2.2)



一. (单位)冲激响应 $h(t)$

1. 定义: 输入信号为 $\delta(t)$ 时的零状态响应。



例 $y''(t) + 5y'(t) + 6y(t) = f(t) \quad (1)$

$$f(t) \rightarrow \delta(t), y(t) \rightarrow h(t)$$

$$h''(t) + 5h'(t) + 6h(t) = \delta(t) \quad (2)$$

$$h''(t) + 5h'(t) + 6h(t) = 0, t > 0 \quad (3)$$

2. $h(t)$ 具有齐次解的形式

$$\Rightarrow h(t) = (C_1 e^{-2t} + C_2 e^{-3t}) \underline{\varepsilon(t)} \quad (4)$$

3. $h(t)$ 的求取 “匹配系数法”



例2.2-1 $y''(t) + 5y'(t) + 6y(t) = f(t) \quad (1)$

$$h''(t) + 5h'(t) + 6h(t) = \delta(t) \quad (2)$$

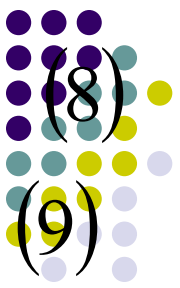
$$\Rightarrow h(t) = (C_1 e^{-2t} + C_2 e^{-3t}) \cdot \varepsilon(t) \quad (4)$$

$$h'(t) = (C_1 + C_2)\delta(t) + (-2C_1 e^{-2t} - 3C_2 e^{-3t}) \cdot \varepsilon(t) \quad (5)$$

$$h''(t) = (C_1 + C_2)\delta'(t) + (4C_1 e^{-2t} + 9C_2 e^{-3t})\varepsilon(t) + (-2C_1 - 3C_2)\delta(t) \quad (6)$$

$$\begin{cases} (C_1 + C_2) = 0 \\ 5(C_1 + C_2) + (-2C_1 - 3C_2) = 1 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = -1 \end{cases} \Rightarrow$$

$$h(t) = (e^{-2t} - e^{-3t}) \cdot \underline{\varepsilon(t)} \quad (7)$$



例 2.2-2 求 $h(t)$ $y''(t) + 5y'(t) + 6y(t) = 2f'(t) + 3f(t)$ (8)

(a). “匹配系数法” $h''(t) + 5h'(t) + 6h(t) = 2\delta'(t) + 3\delta(t)$ (9)

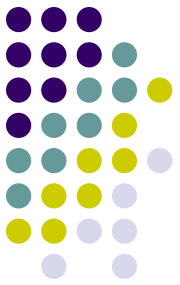
令: $h(t) = (c_1 e^{-2t} + c_2 e^{-3t}) \cdot \varepsilon(t)$ (10)

$$h'(t) = (c_1 + c_2)\delta(t) + (-2c_1 e^{-2t} - 3c_2 e^{-3t}) \cdot \varepsilon(t) \quad (11)$$

$$h''(t) = (c_1 + c_2)\delta'(t) + (-2c_1 - 3c_2)\delta(t) + (4c_1 e^{-2t} + 9c_2 e^{-3t})\varepsilon(t) \quad (12)$$

$$\begin{cases} c_1 + c_2 = 2 \\ 5(c_1 + c_2) + (-2c_1 - 3c_2) = 3 \end{cases} \Rightarrow \begin{cases} c_1 = -1 \\ c_2 = 3 \end{cases}$$

$$h(t) = (3e^{-3t} - e^{-2t})\varepsilon(t) \quad (13)$$



(b) 由线性时不变性质:

$$\underline{y''(t) + 5y'(t) + 6y(t) = f(t)} \quad (1)$$

$$\Rightarrow h_1(t) = (e^{-2t} - e^{-3t}).\varepsilon(t) \quad (7)$$

$$\underline{y''(t) + 5y'(t) + 6y(t) = 2f'(t) + 3f(t)} \quad (8)$$

$$\begin{aligned} \Rightarrow h(t) &= 2h_1'(t) + 3h_1(t) \\ &= (3e^{-3t} - e^{-2t})\varepsilon(t) \quad (13) \end{aligned}$$

二. (单位)阶跃响应



1. 定义 $\xrightarrow[\varepsilon(t)]{f(t)} \boxed{\{x(0) = 0\}} \xrightarrow[g(t)]{y_{zs}(t)}$

2. $g(t)$ 的求取

依据: 线性时不变系统的性质

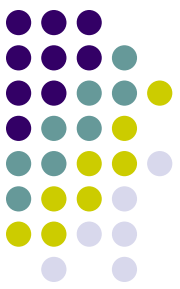
$$\varepsilon(t) = \int_{-\infty}^t \delta(x) dx \Rightarrow g(t) = \int_{-\infty}^t h(x) dx$$

例 求: $g(t)$ $y''(t) + 5y'(t) + 6y(t) = f(t)$ (1)

$$\Rightarrow h_1(t) = (e^{-2t} - e^{-3t}) \cdot \varepsilon(t) \quad (7)$$

$$g(t) = \int_{-\infty}^t (e^{-2x} - e^{-3x}) \varepsilon(x) dx$$

$$g(t) = \left(\frac{1}{3} e^{-3t} - \frac{1}{2} e^{-2t} + \frac{1}{6} \right) \underline{\varepsilon(t)} \quad (14)$$



例 求(8)式所示系统的阶跃响应.

$$y''(t) + 5y'(t) + 6y(t) = 2f'(t) + 3f(t) \quad (8)$$

解:

$$y_1''(t) + 5y_1'(t) + 6y_1(t) = f(t) \quad (1)$$

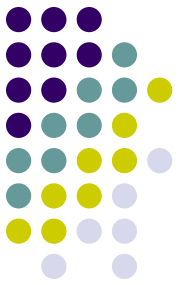
$$\Rightarrow h_1(t) = (e^{-2t} - e^{-3t}) \cdot \varepsilon(t) \quad (7)$$

$$\Rightarrow h(t) = 2h_1'(t) + 3h_1(t)$$

$$= (3e^{-3t} - e^{-2t}) \varepsilon(t) \quad (13)$$

$$g(t) = \int_{-\infty}^t h(x) dx$$

$$= \left(\frac{1}{2} + \frac{1}{2} e^{-2t} - e^{-3t} \right) \cdot \underline{\varepsilon(t)}$$

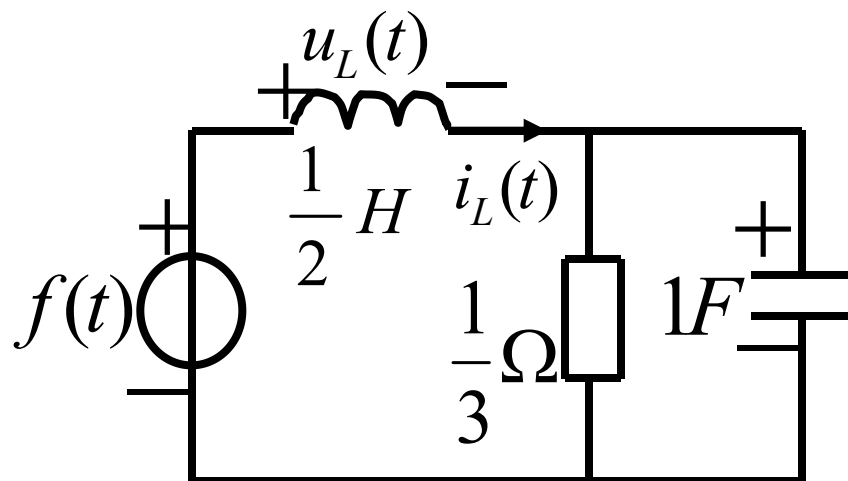


2.3 LTI连续系统的响应 (§ 2.1)

- 一. 微分方程的编写
- 二. 零输入响应
- 三. 零状态响应
- 四. 关于初始值的说明
- 五. 系统的全响应
- 六. 连续LTI系统时域分析

一. 微分方程的编写

例



已知: $u_C(0_-) = 2V$,
 $i_L(0_-) = 1A$,
 $f(t) = 1V, t > 0$
 $y_1(t) = u_C(t)$
 $y_2(t) = i_L(t)$
 $y_3(t) = u_L(t)$,

求: 列出分别以 $y_1(t)$, $y_2(t)$, $y_3(t)$ 为输出的微分方程

解: $y_1''(t) + 3y_1'(t) + 2y_1(t) = 2f(t) \quad (1)$

$$y_2''(t) + 3y_2'(t) + 2y_2(t) = 6f(t) + 2f'(t) \quad (2)$$

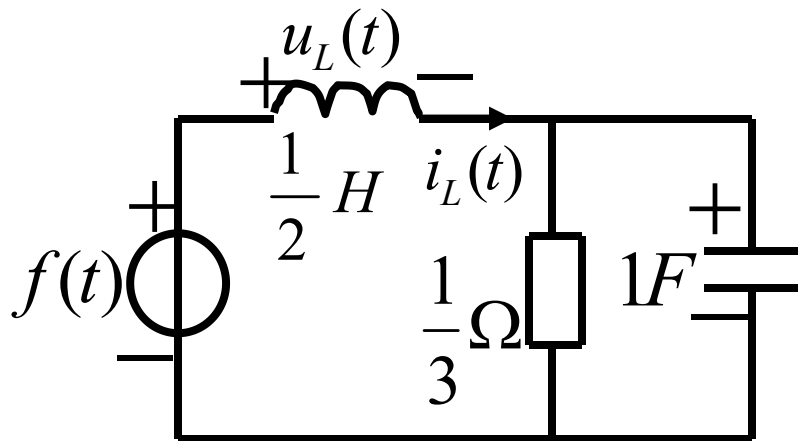
$$y_3''(t) + 3y_3'(t) + 2y_3(t) = f''(t) + 3f'(t) \quad (3)$$



二. 零输入响应 $y_{zi}(t)$

1. 定义：系统的输入为零，完全由系统的初始状态引起的响应。

例



已知：

$$u_c(0_-) = 2V,$$
$$i_L(0_-) = 1A,$$
$$f(t) = 0, t \geq 0$$
$$y(t) = i_L(t)$$

$$y''(t) + 3y'(t) + 2y(t) = 6f(t) + 2f'(t) \quad (1)$$

$$y_{zi}''(t) + 3y_{zi}'(t) + 2y_{zi}(t) = 0 \quad (2)$$

2. 求解：

$$y(0_-) = y_{zi}(0) = 1, y'(0_-) = y_{zi}'(0) = -4 \quad (3)$$

$$y_{zi}(t) = C_1 e^{-2t} + C_2 e^{-t} \quad (4) \Rightarrow C_1 = 3, C_2 = -2$$

$$y_{zi}(t) = 3e^{-2t} - 2e^{-t} \quad t \geq 0 \quad (5)$$

例2.1-4 求零输入响应.

$$y(0_-) = 1, y'(0_-) = 5$$

$$y''(t) + 5y'(t) + 4y(t) = 2f'(t) - 4f(t) \quad (1)$$

解: $y''_{zi}(t) + 5y'_{zi}(t) + 4y_{zi}(t) = 0 \quad (2)$

$$\lambda^2 + 5\lambda + 4 = 0 \quad (3) \quad \lambda_1 = -1 \quad \lambda_2 = -4$$

$$y_{zi}(t) = C_1 e^{-t} + C_2 e^{-4t} \quad (4)$$

$$\begin{cases} C_1 + C_2 = 1 \\ -C_1 - 4C_2 = 5 \end{cases} \quad (5)$$

$$\begin{cases} C_1 = 3 \\ C_2 = -2 \end{cases} \quad (6)$$

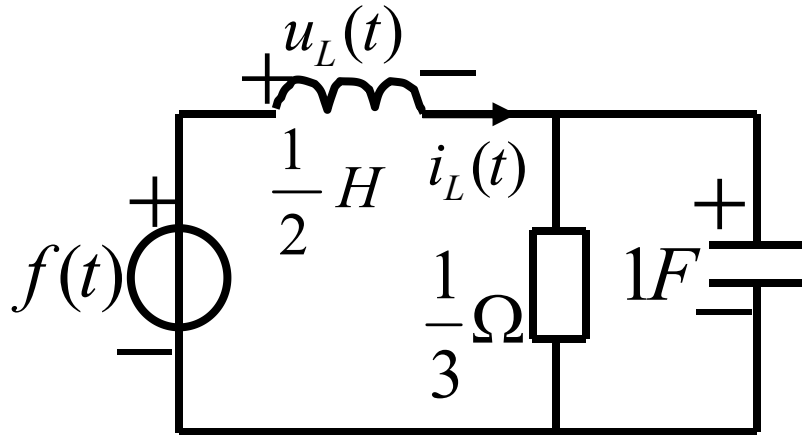
$$y_{zi}(t) = 3e^{-t} - 2e^{-4t}, \quad t \geq 0 \quad (7)$$



三. 零状态响应 $y_{zs}(t)$

1. 定义: 系统的初始状态为零, 完全由系统的输入引起的响应.

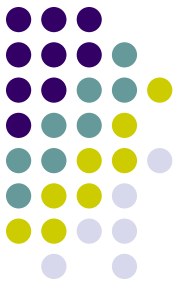
例



已知: $u_c(0_-) = 0V$,
 $i_L(0_-) = 0A$,
 $f(t) = 1V, t \geq 0$
 $y(t) = i_L(t)$

$$y''(t) + 3y'(t) + 2y(t) = 6f(t) + 2f'(t)$$





2.证明: $f(t) \longrightarrow \boxed{\{x(0) = 0\}} \longrightarrow y_{zs}(t)$

$$\delta(t) \longrightarrow h(t)$$

$$\delta(t - \tau) \longrightarrow h(t - \tau)$$

$$f(\tau)\delta(t - \tau) \longrightarrow f(\tau)h(t - \tau)$$

$$\int_{-\infty}^{+\infty} f(\tau)\delta(t - \tau)d\tau \longrightarrow \int_{-\infty}^{+\infty} f(\tau)h(t - \tau)d\tau$$

$$f(t) * \delta(t) \longrightarrow f(t) * h(t)$$

$$f(t) \longrightarrow y_{zs}(t)$$

$$y_{zs}(t) = f(t) * h(t)$$

例 已知系统: $y''(t) + 3y'(t) + 2y(t) = f(t)$ 求: $y_{zs}(t)$

$$f(t) = \varepsilon(t)$$

$$h''(t) + 3h'(t) + 2h(t) = \delta(t)$$

$$h(t) = [e^{-t} - e^{-2t}] \varepsilon(t)$$

$$y_{zs}(t) = [e^{-t} - e^{-2t}] \varepsilon(t) * \varepsilon(t)$$

$$y_{zs}(t) = \left[\frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \right] \varepsilon(t)$$



例 2.1-5 已知: $y''(t) + 5y'(t) + 4y(t) = 2f'(t) - 4f(t)$

$f(t) = \varepsilon(t)$ 求: $y_{zs}(t)$

$$h''(t) + 5h'(t) + 4h(t) = 2\delta'(t) - 4\delta(t)$$

$$h(t) = [C_1 e^{-t} - C_2 e^{-4t}] \varepsilon(t)$$

$$h'(t) = [C_1 + C_2] \delta(t) + [-C_1 e^{-t} - 4C_2 e^{-4t}] \varepsilon(t)$$

$$h''(t) = [C_1 + C_2] \delta'(t) + [-C_1 - 4C_2] \delta(t) + [C_1 e^{-t} + 16C_2 e^{-4t}] \varepsilon(t)$$

$$\begin{cases} C_1 + C_2 = 2 \\ (-C_1 - 4C_2) + 5(C_1 + C_2) = -4 \end{cases} \quad \begin{cases} C_1 = -2 \\ C_2 = 4 \end{cases}$$

$$h(t) = [-2e^{-t} + 4e^{-4t}] \varepsilon(t)$$

$$y_{zs}(t) = f(t) * h(t) = [-2e^{-t} + 4e^{-4t}] \varepsilon(t) * \varepsilon(t)$$

$$= [2e^{-t} - e^{-4t} - 1] \varepsilon(t)$$



四. 关于初始值的说明



1. $y(0_-)$: 不考虑输入信号的作用, 仅由初始状态产生的输出响应在 $t=0$ 时刻的值, 即零输入响应的初始值:

$$y(0_-) = y_{zi}(0) = y_x(0)$$

2. $y(0_+)$: 考虑输入信号及初始状态的共同作用下, 输出响应在 $t=0$ 时刻的值, 即全响应的初始值:

$$y(0_+) = y_{zi}(0) + y_{zs}(0)$$

五. 全响应

$$y(t) = y_{zi}(t) + y_{zs}(t) = y_x(t) + y_f(t)$$

$$\begin{cases} y_{zi}''(t) + a_1 y_{zi}'(t) + a_0 y_{zi}(t) = 0 \\ y_{zi}(0) = y(0_-) \quad y_{zi}'(0) = y'(0_-) \end{cases}$$

$$y_{zs}(t) = y_f(t) = f(t) * h(t)$$

$$h(t) = [C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}] \varepsilon(t) \quad \text{比较系数法确定 } C_1, C_2$$

例 已知系统 $y''(t) + 4y'(t) + 3y(t) = f(t)$ [ex.2.4(1)]

求全响应 $f(t) = \varepsilon(t), y'(0_-) = y(0_-) = 1$

(1). $y_{zi}(t)$ $y_{zi}''(t) + 4y_{zi}'(t) + 3y_{zi}(t) = 0$

$$y_{zi}(t) = A_1 e^{-t} + A_2 e^{-3t} \quad y_{zi}'(0) = y_{zi}(0) = 1$$

$$y_{zi}(t) = 2e^{-t} - e^{-3t}, \quad t \geq 0$$

(2). $y_{zs}(t)$ $y_{zs}(t) = f(t) * h(t)$

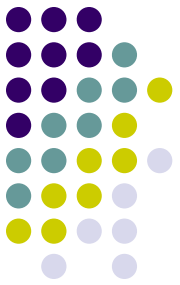
$$h''(t) + 4h'(t) + 3h(t) = \delta(t)$$

$$h(t) = [C_1 e^{-t} + C_2 e^{-3t}] \varepsilon(t) = 0.5 [e^{-t} - e^{-3t}] \varepsilon(t)$$

$$y_{zs}(t) = 0.5 [e^{-t} - e^{-3t}] \varepsilon(t) * \varepsilon(t) = \left[\frac{1}{3} - \frac{1}{2} e^{-t} + \frac{1}{6} e^{-3t} \right] \varepsilon(t)$$

(3). $y(t)$ $y(t) = y_{zi}(t) + y_{zs}(t) = \frac{1}{3} + \frac{3}{2} e^{-t} - \frac{5}{6} e^{-3t}, \quad t \geq 0$





六. 连续LTI系统时域分析



线性常系数微分方程

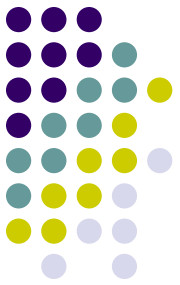
模拟框图

(卷积运算)

$$\begin{cases} \text{经典解法} & y_h(t) + y_p(t) \\ \text{时域解法} & y_{zs}(t) + y_{zi}(t) \end{cases}$$

$$\begin{cases} y_{zi}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ C_1, C_2 \text{ 由与 } f(t) \text{ 无关的初始值决定} \\ y_{zi}(0) = y(0_-), y'_{zi}(0) = y'(0_-) \end{cases}$$

$$\begin{cases} y_{zs}(t) = y_f(t) = f(t) * h(t) \\ h(t) \text{ 的求取} & \text{匹配系数法} \end{cases}$$



第三章 离散系统的时域分析

要点:

1. $\delta(k), \varepsilon(k)$

2. $h(k), g(k)$

3. 卷积和 $f_1(k) * f_2(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i)$

4. 离散系统时域分析 $y(k) = y_{zi}(k) + y_{zs}(k)$
 $= y_x(k) + y_f(k)$

3.1 离散信号与系统概述 (§ 1.1 § 3.2)

一. 时域分析



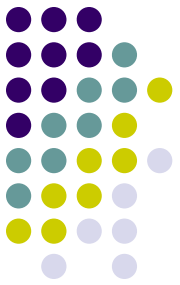
线性常系数微分方程

模拟框图

(卷积运算)

$$\begin{cases} \text{经典解法} & y_h(t) + y_p(t) \\ \text{时域解法} & y_{zs}(t) + y_{zi}(t) \end{cases}$$

$$\begin{cases} y_{zi}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ C_1, C_2 \text{ 由与 } f(t) \text{ 无关的初始值决定} \\ y_{zs}(t) = y_f(t) = f(t) * h(t) \\ h(t) \text{ 的求取} \quad \text{“匹配系数法”} \end{cases}$$



线性常系数差分方程

$$\begin{cases} \text{经典解法} & y_h(k) + y_p(k) \\ \text{时域解法} & y_{zs}(k) + y_{zi}(k) \end{cases}$$

模拟框图

(卷积和运算)

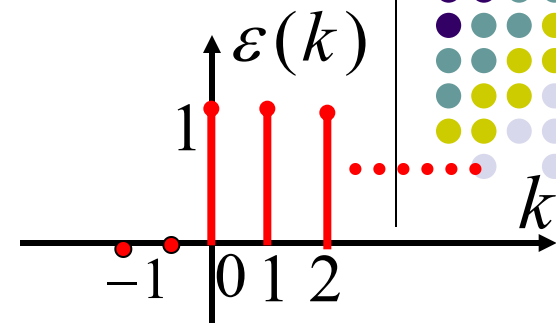
$$\begin{cases} y_{zs}(k) = C_1 \lambda_1^k + C_2 \lambda_2^k \\ C_1, C_2 \text{ 由与 } f(k) \text{ 无关的初始值决定} \end{cases}$$

$$\begin{cases} y_{zs}(k) = f(k) * h(k) \\ h(k) \text{ 的求取 “迭代法求初值”} \end{cases}$$

二. 基本离散信号(序列)

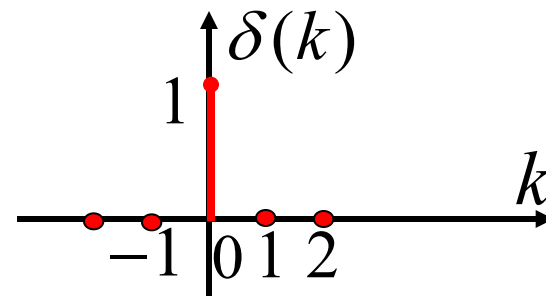
1. 单位阶跃序列

$$\varepsilon(k) = \begin{cases} 1, & \dots k \geq 0 \\ 0, & \dots k < 0 \end{cases}$$



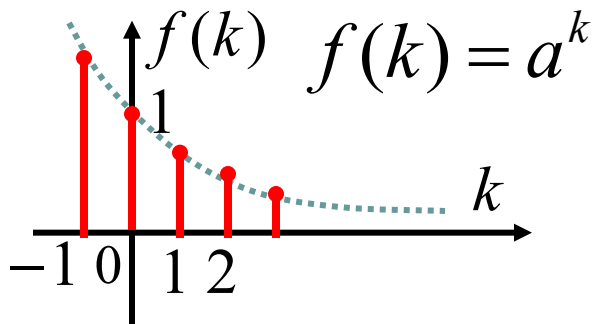
2. 单位(冲激)序列

$$\delta(k) = \begin{cases} 1, & \dots k = 0 \\ 0, & \dots k \neq 0 \end{cases}$$

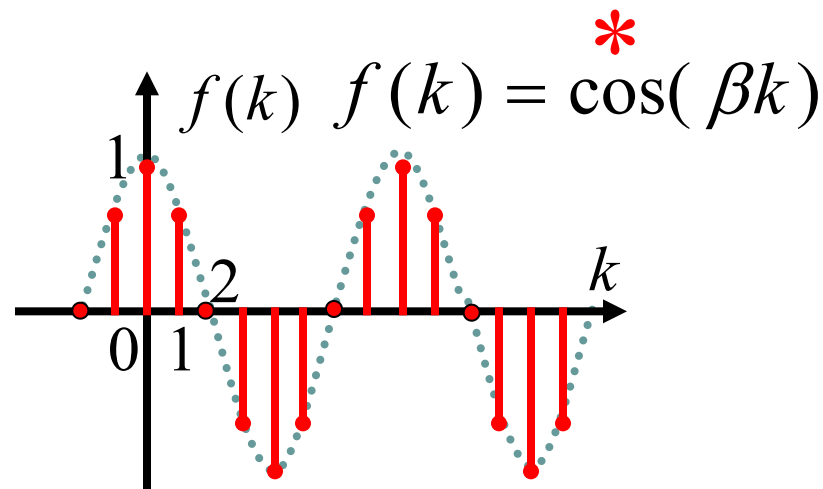


3. 指数序列

$$f(k) = a^k$$



4. 正弦序列 $f(k) = \cos(\beta k)$



三. 单位序列的性质

$$1. \quad \delta(k) = \delta(-k)$$

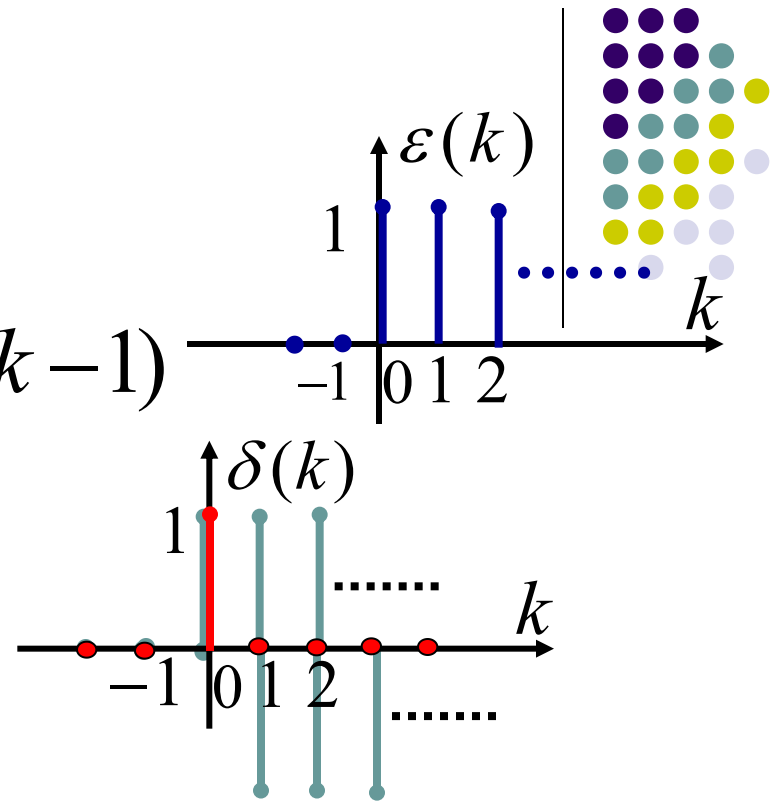
$$2. \quad \delta(k) = \nabla \varepsilon(k) = \varepsilon(k) - \varepsilon(k-1)$$

$$3. \quad \varepsilon(k) = \sum_{j=-\infty}^k \delta(j)$$

$$4. \quad f(k) \cdot \delta(k-N) = f(N) \cdot \delta(k-N)$$

$$5. \quad \sum_{k=-\infty}^{\infty} f(k) \cdot \delta(k-N) = f(N)$$

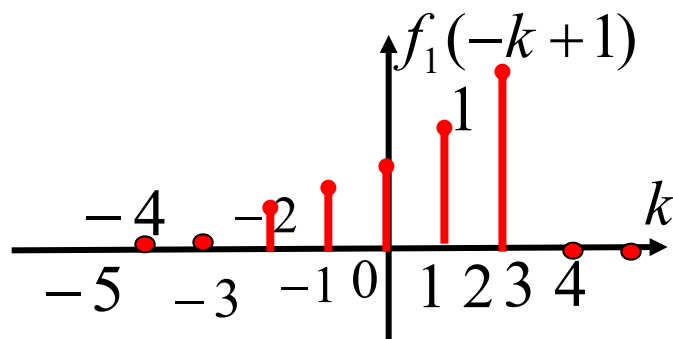
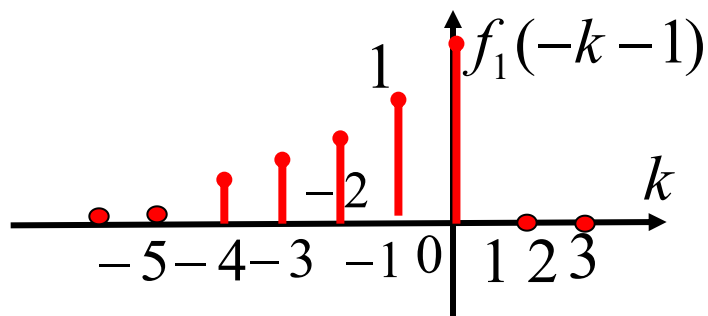
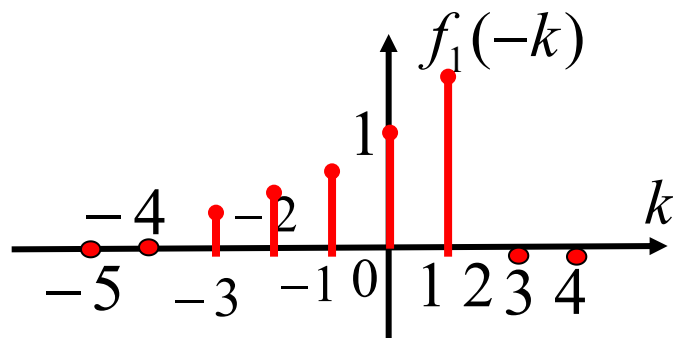
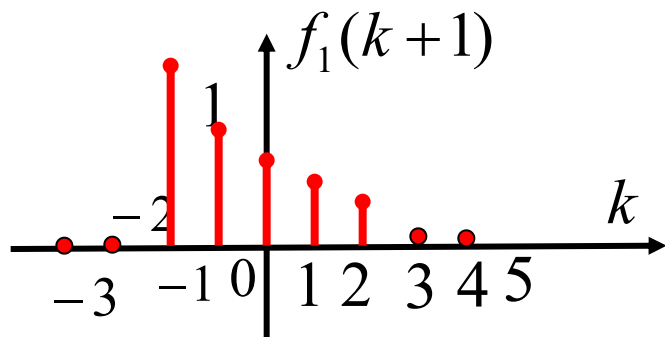
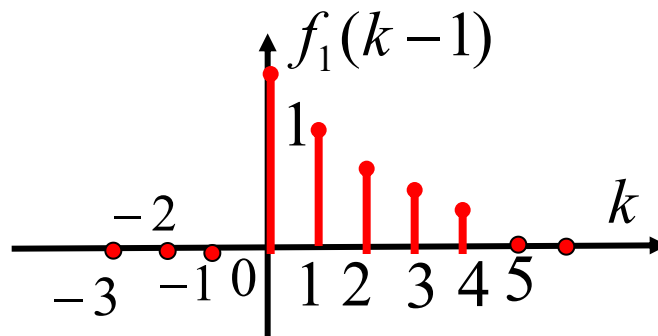
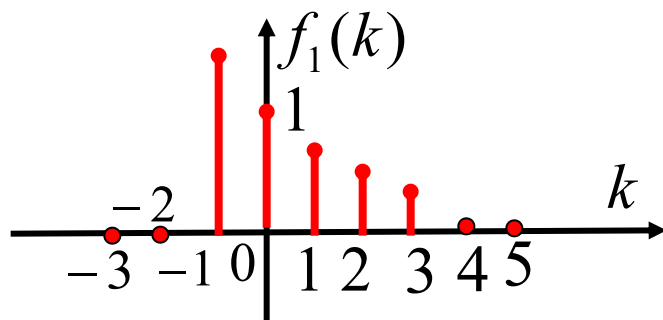
$$6. \quad f(k) * \delta(k) = f(k) \quad f(k) * \delta(k-N) = f(k-N)$$



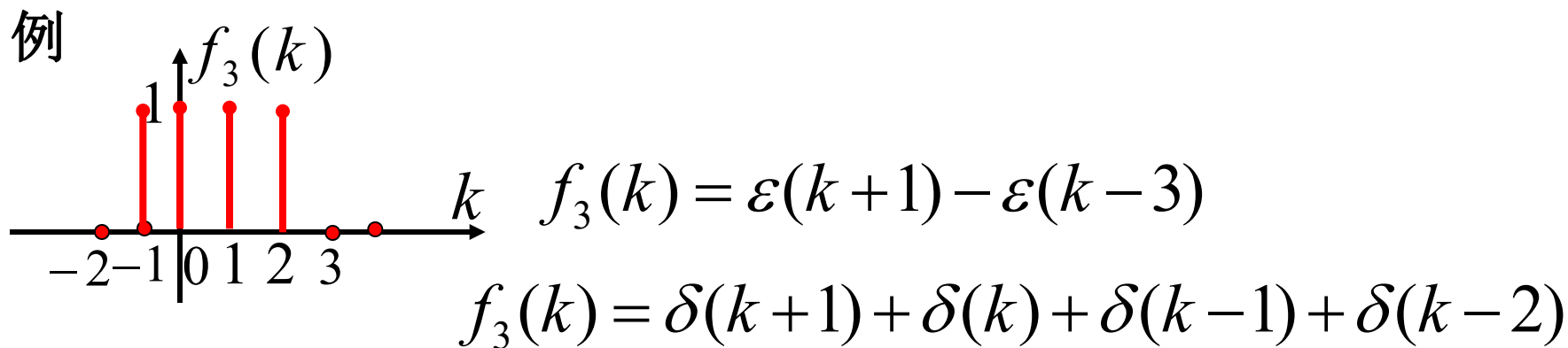
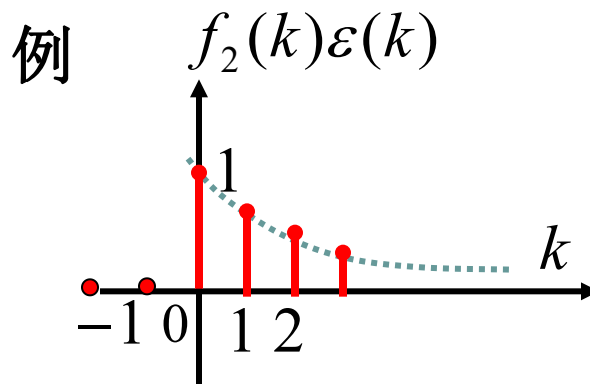
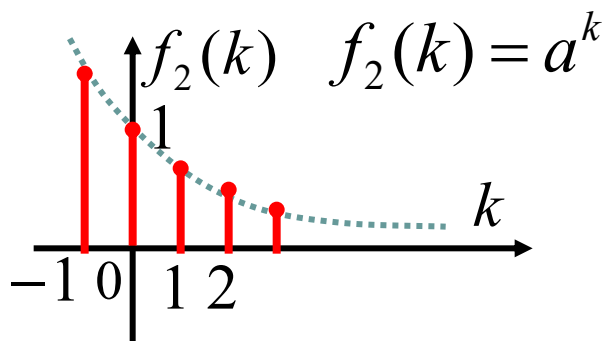
四. 序列的基本运算



1. 反折和平移:



2. 基本函数的使用



五. 离散系统描述—差分方程

1. 序列的差分

$$\nabla f(k) = f(k) - f(k-1) \quad \Delta f(k) = f(k+1) - f(k)$$



例. 求: $\nabla f(k)$, $ex.3.1(p.110)$

$$f(k) = \begin{cases} 0, & k < 0 \\ (\frac{1}{2})^k, & k \geq 0 \end{cases}$$

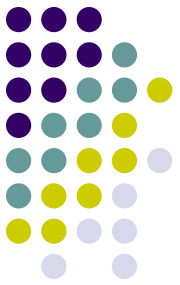
$$\nabla f(k) = f(k) - f(k-1)$$

$$0, \quad k < 0$$

$$= \begin{cases} 1, & k = 0 \\ (\frac{1}{2})^k - (\frac{1}{2})^{k-1} = -(\frac{1}{2})^k, & k \geq 1 \end{cases}$$

$$\nabla f(0) = f(0) - f(-1)$$





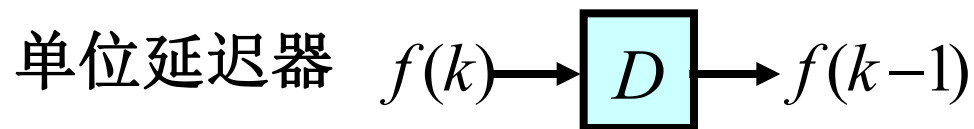
2. 差分方程

例.ex.3.23(P.113)贷款 M 万元,月利率 $\beta = 1\%$,定期月初还款 $f(k)$ 万元,贷款余额 $y(k)$ 万元,建立差分方程.

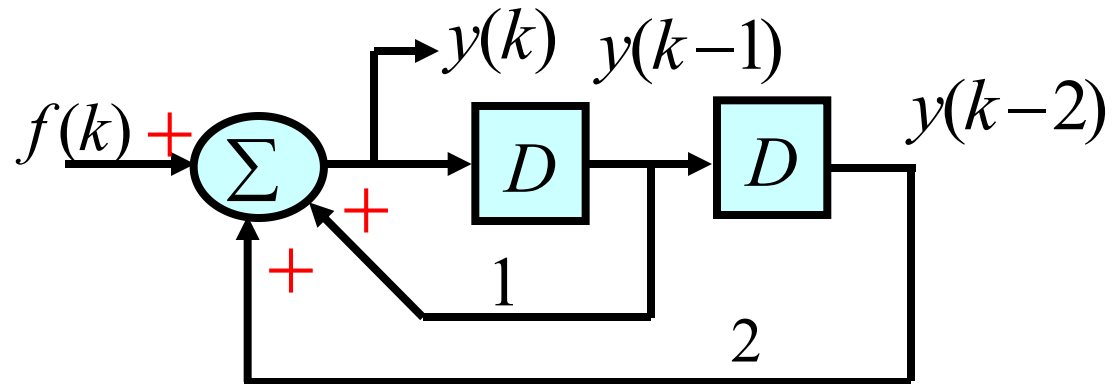
$$y(k) = (1 + \beta)y(k-1) - f(k)$$

$$y(k) - (1 + \beta)y(k-1) = -f(k)$$

3. 离散系统的框图描述



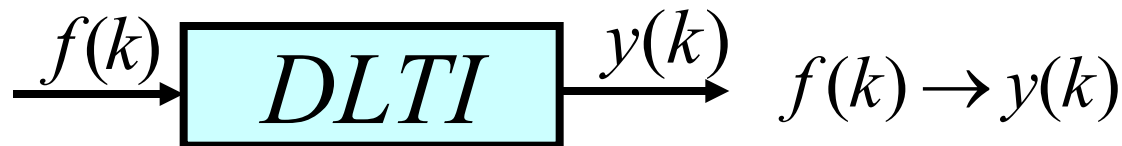
例 3.2-1 求: 差分方程



解:

$$y(k) = y(k-1) + 2y(k-2) + f(k) \quad y(k) - y(k-1) - 2y(k-2) = f(k)$$

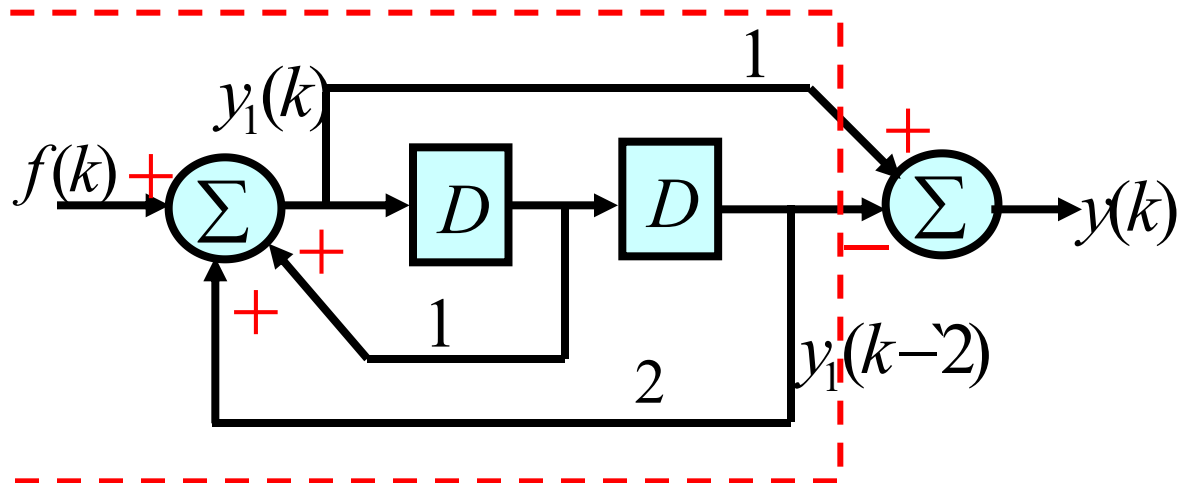
4. 离散LTI系统的主要性质



(1) 线性: $a_1 f_1(k) + a_2 f_2(k) \rightarrow a_1 y_1(k) + a_2 y_2(k)$

(2) 时移不变性: $f(k-N) \rightarrow y(k-N)$

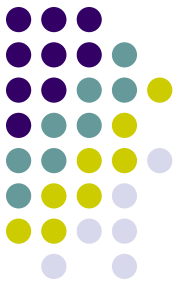
例 3.2-2 求:
差分方程



解:

$$y_1(k) - y_1(k-1) - 2y_1(k-2) = f(k) \quad y(k) = y_1(k) - y_1(k-2)$$

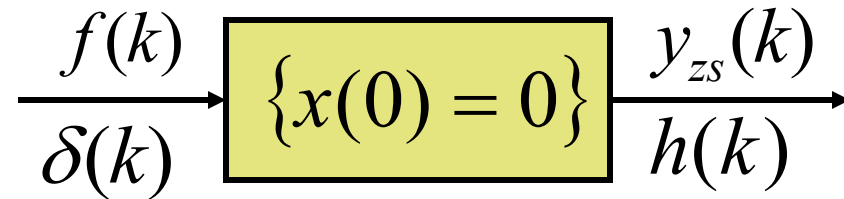
$$y(k) - y(k-1) - 2y(k-2) = f(k) - f(k-2)$$



3.2 单位序列响应和单位阶跃响应 (§ 3.2)

一. 单位序列响应

1. 定义



2. $h(k)$ 的形式

$$y(k) + a_1 y(k-1) + a_0 y(k-2) = f(k)$$

$$h(k) + a_1 h(k-1) + a_0 h(k-2) = \delta(k)$$

$$\Rightarrow \lambda^2 + a_1 \lambda + a_0 = 0 \Rightarrow \lambda_1, \lambda_2$$

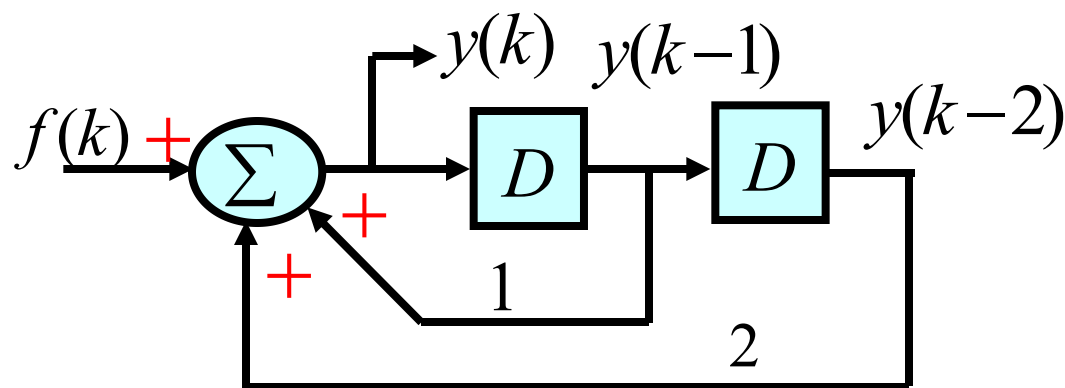
$$\begin{cases} h(k) = (C_1 \lambda_1^k + C_2 \lambda_2^k) \underline{\varepsilon(k)} * \\ h(0), h(1) \end{cases}$$

3. $h(k)$ 的求取

迭代法求初值

例 3.2-1 求: $h(k)$

解:



$$y(k) - y(k-1) - 2y(k-2) = f(k)$$

$$h(k) - h(k-1) - 2h(k-2) = \delta(k)$$

$$h(0) = h(-1) + 2h(-2) + \delta(k) = 1$$

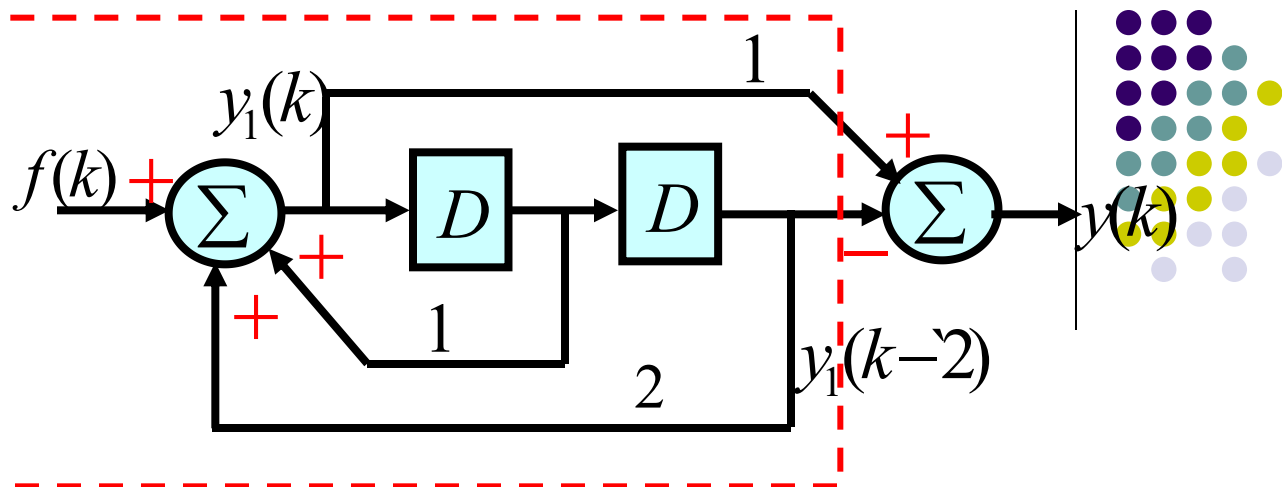
$$h(1) = h(0) + 2h(-1) + \delta(1) = 1$$

$$\lambda^2 - \lambda - 2 = 0 \Rightarrow \lambda_1 = -1, \lambda_2 = 2 \quad h(k) = [C_1(-1)^k + C_2(2)^k] \varepsilon(k)$$

$$\begin{cases} C_1 + C_2 = 1 \\ C_1(-1) + C_2(2) = 1 \end{cases} \quad C_1 = \frac{1}{3}, C_2 = \frac{2}{3}$$

$$h(k) = \left[\frac{1}{3}(-1)^k + \frac{2}{3}(2)^k \right] \varepsilon(k)$$

例 3.2-2 求: $h(k)$



解:

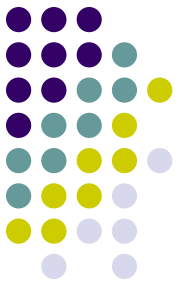
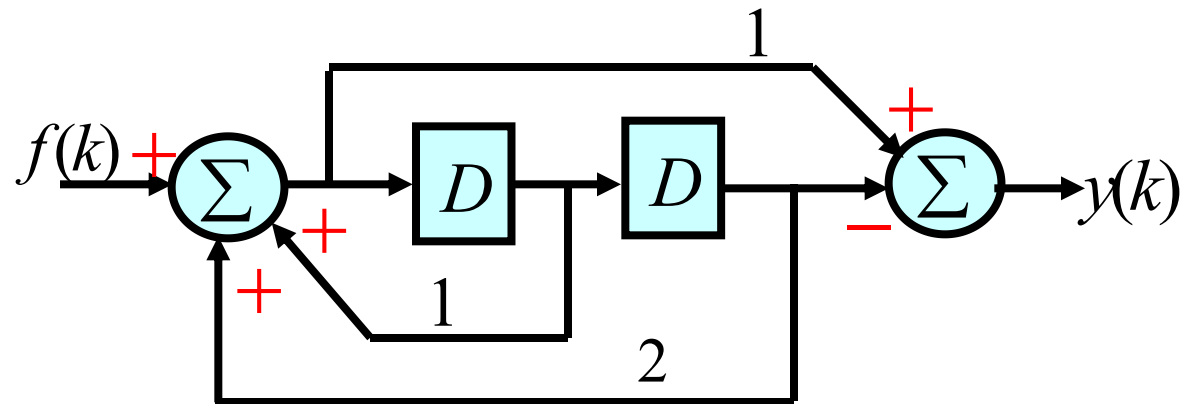
$$y_1(k) - y_1(k-1) - 2y_1(k-2) = f(k)$$

$$h_1(k) = \left[\frac{1}{3}(-1)^k + \frac{2}{3}(2)^k \right] \varepsilon(k)$$

$$y(k) - y(k-1) - 2y(k-2) = f(k) - f(k-2)$$

$$h(k) = h_1(k) - h(k-2)$$

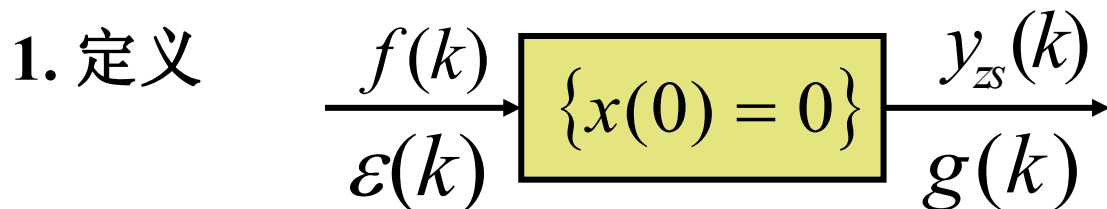
$$h(k) = \left[\frac{1}{3}(-1)^{\underline{k}} + \frac{2}{3}(2)^{\underline{k}} \right] \underline{\varepsilon(k)} - \left[\frac{1}{3}(-1)^{\underline{k-2}} + \frac{2}{3}(2)^{\underline{k-2}} \right] \underline{\varepsilon(k-2)}$$



$$h(k) = \left[\frac{1}{3}(-1)^{\underline{k}} + \frac{2}{3}(2)^{\underline{k}} \right] \underline{\varepsilon(k)} - \left[\frac{1}{3}(-1)^{\underline{k-2}} + \frac{2}{3}(2)^{\underline{k-2}} \right] \underline{\varepsilon(k-2)}$$

$$= \begin{cases} 0, & k < 0 \\ \frac{1}{3}(-1)^k + \frac{2}{3}(2)^k, & k = 0, 1 \\ \frac{1}{2}(2)^k, & k \geq 2 \end{cases}$$

二. 单位阶跃响应



2. $g(k)$ 的求取

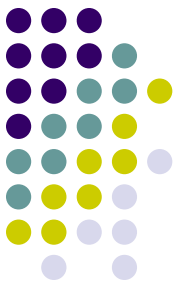
$$\varepsilon(k) = \sum_{i=-\infty}^k \delta(i) \Rightarrow g(k) = \sum_{i=-\infty}^k h(i)$$

例3.2-3 求 $g(k)$ $y(k) - y(k-1) - 2y(k-2) = f(k)$ (1)

$$h(k) = \left[\frac{1}{3}(-1)^k + \frac{2}{3}(2)^k \right] \varepsilon(k)$$

$$\begin{aligned} g(k) &= \left[\frac{1}{3} \sum_{i=0}^k (-1)^i + \frac{2}{3} \sum_{i=0}^k (2)^i \right] \cdot \varepsilon(k) \\ &= \left[\frac{1}{6}(-1)^k + \frac{4}{3}(2)^k - \frac{1}{2} \right] \varepsilon(k) \end{aligned}$$





3.3 卷积和 (§ 3.3)

一. 序列的卷积和

定义: $f_1(k), f_2(k), -\infty < k < \infty$

$$f_1(k) * f_2(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i)$$

例3.3-1(2) $f_1(k) = (0.5)^k \varepsilon(k), f_3(k) = \varepsilon(k)$

计算: $y(k) = f_1(k) * f_3(k)$

$$y(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i) = \sum_{i=-\infty}^{\infty} (0.5)^i \varepsilon(i) \varepsilon(k-i)$$

$$= \left[\sum_{i=0}^k (0.5)^i \right] \varepsilon(k) = \frac{1 - (0.5)^{k+1}}{1 - 0.5} \varepsilon(k) = 2[1 - (0.5)^{k+1}] \varepsilon(k)$$

二. 卷积和的性质



$$(1) \quad f_1(k) * f_2(k) = f_2(k) * f_1(k)$$

$$(2) \quad f_1(k) * [f_2(k) + f_3(k)] = f_1(k) * f_2(k) + f_1(k) * f_3(k)$$

$$(3) \quad f_1(k) * [f_2(k) * f_3(k)] = [f_1(k) * f_2(k)] * f_3(k)$$

$$(4) \quad f(k) * \delta(k - N) = f(k - N) *$$

$$a\delta(k - N_1) * b\delta(k - N_2) = (a \times b)\delta(k - N_1 - N_2) *$$

$$x_1(k - N_1) * x_2(k - N_2) = x_1(k) * x_2(k) * \delta(k - N_1 - N_2)$$

三. 卷积和的计算



(1) 定义 例 计算卷积和 $y(k) = \varepsilon(k) * \varepsilon(k)$

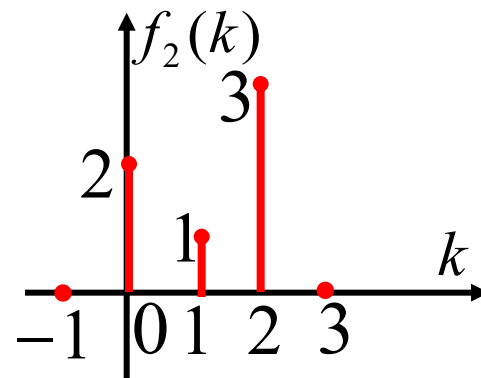
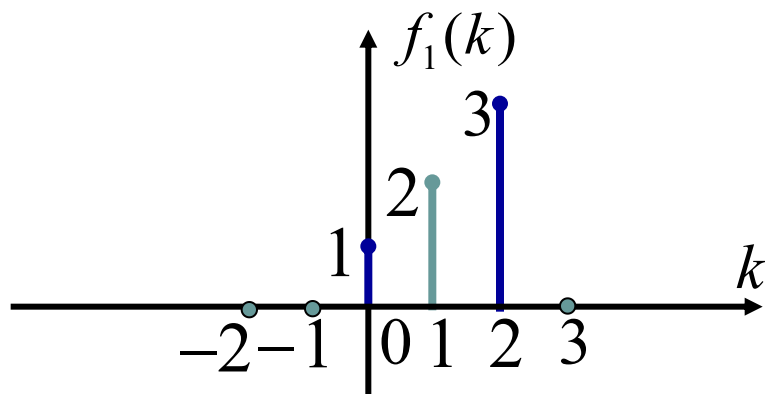
解:
$$y(k) = \sum_{i=-\infty}^{\infty} \varepsilon(i) \varepsilon(k-i)$$
$$= \left[\sum_{i=0}^k 1 \right] \cdot \varepsilon(k) = (k+1) \cdot \varepsilon(k)$$

例 计算卷积和 $y(k) = [a^k \varepsilon(k)] * [b^k \varepsilon(k)]$

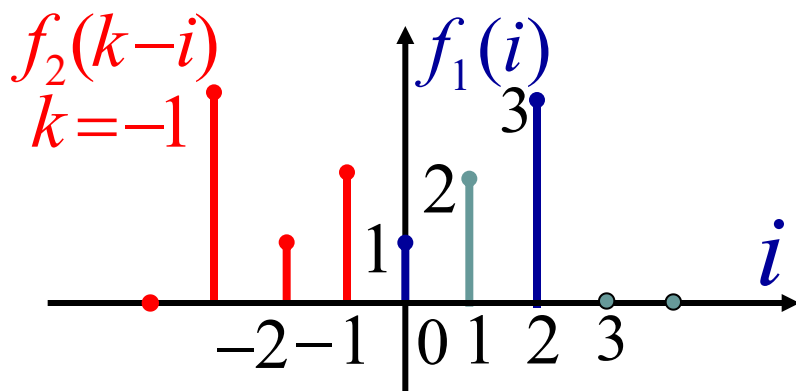
解:
$$y(k) = \sum_{i=-\infty}^{\infty} [a^i \varepsilon(i)] [b^{k-i} \varepsilon(k-i)]$$
$$= \left[\sum_{i=0}^k a^i b^{k-i} \right] \varepsilon(k) = \frac{a^{k+1} - b^{k+1}}{a - b} \varepsilon(k)$$

(2) 图解

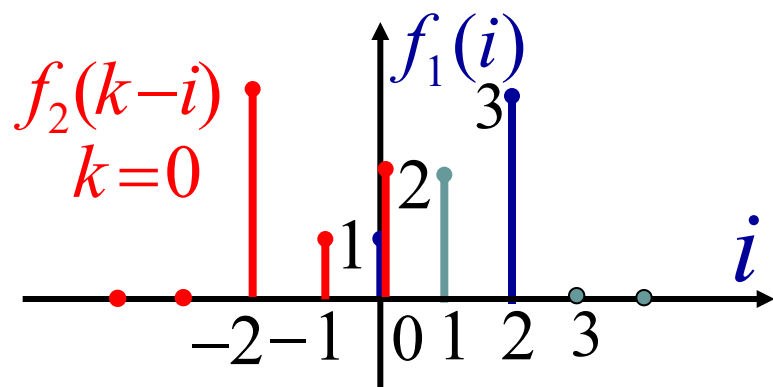
例



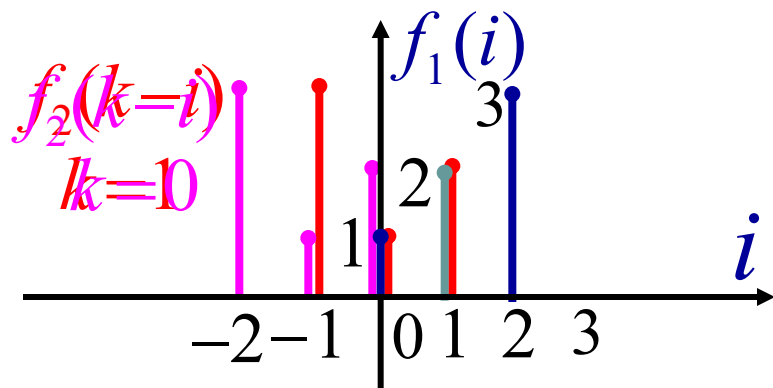
$$f_1(k) * f_2(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i) \quad y(k) = f_1(k) * f_2(k)$$



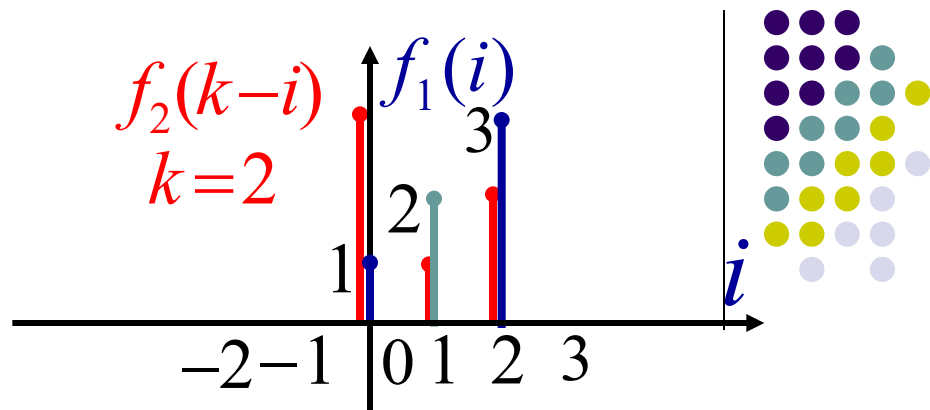
$$y(k) = 0, k \leq -1$$



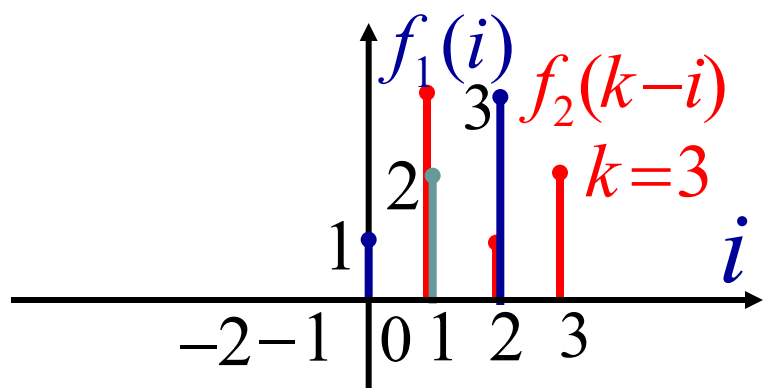
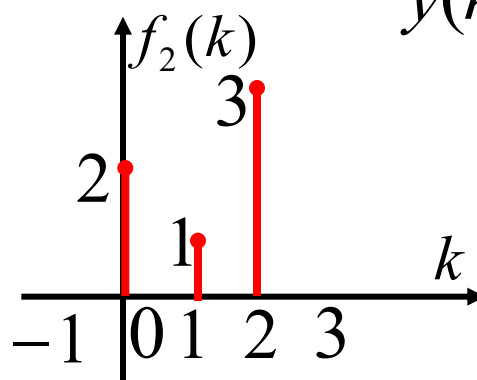
$$y(k) = 2, k = 0$$



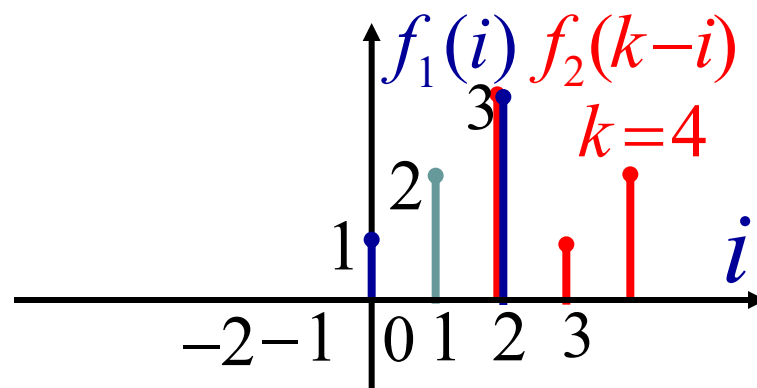
$$y(k) = 5, k = 1$$



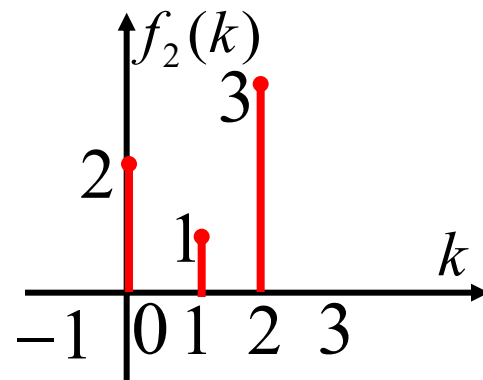
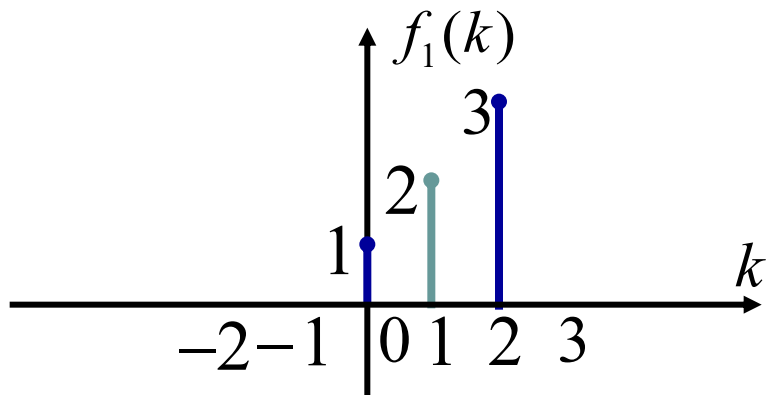
$$y(k) = 11, k = 2$$



$$y(k) = 9, k = 3$$

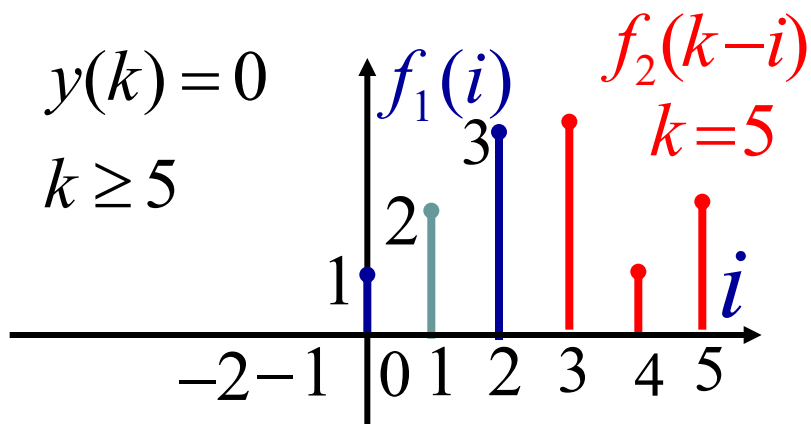
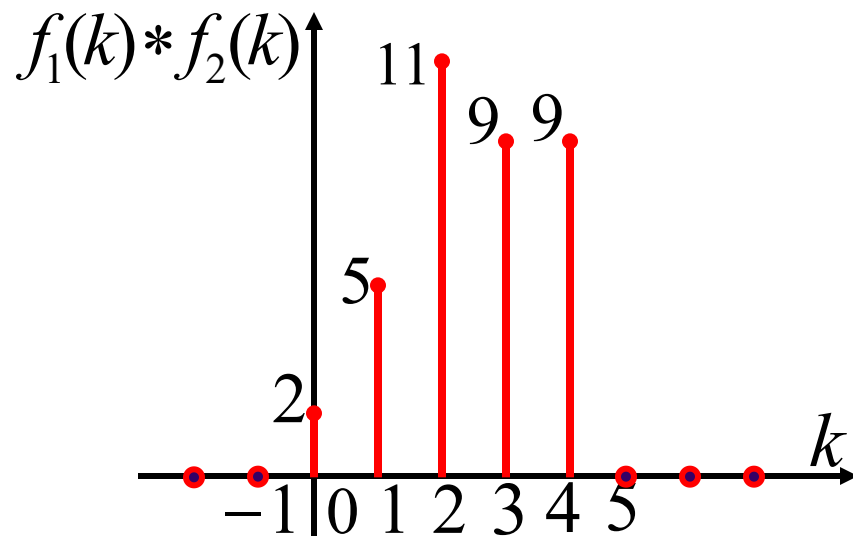


$$y(k) = 9, k = 4$$



$$f_1(k) = \delta(k) + 2\delta(k-1) + 3\delta(k-2)$$

$$f_2(k) = 2\delta(k) + \delta(k-1) + 3\delta(k-2)$$



$$y(k) = 2\delta(k) + 5\delta(k-1) + 11\delta(k-2) + 9\delta(k-3) + 9\delta(k-4)$$

(3) “矩阵表” $a\delta(k-n)*b\delta(k-m)=(a\times b)\delta(k-n-m)$

例 $f_1(k) = \delta(k) + 2\delta(k-1) + 3\delta(k-2)$

$f_2(k) = 2\delta(k) + \delta(k-1) + 3\delta(k-2)$

$f_1(k) * f_2(k) =$

$2\times 1\delta(k) + 2\times 2\delta(k-1) + 2\times 3\delta(k-2)$

$+ 1\times 1\delta(k-1) + 1\times 2\delta(k-2) + 1\times 3\delta(k-3)$

$+ 3\times 1\delta(k-2) + 3\times 2\delta(k-3) + 3\times 3\delta(k-4)$

$2\delta(k) + 5\delta(k-1) + 11\delta(k-2) + 9\delta(k-3) + 9\delta(k-4)$

1 2 3

2 1 3

2 4 6

1 2 3

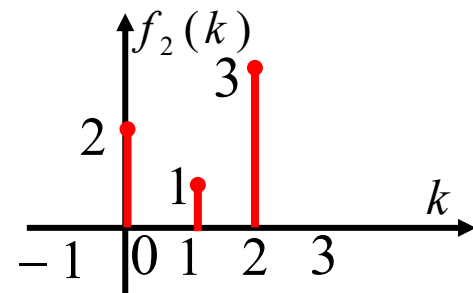
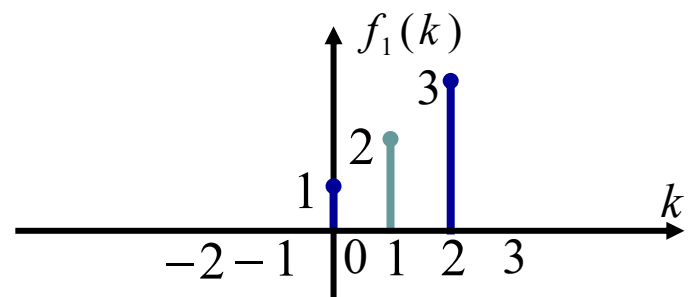
3 6 9

2 5 11 9 9

$k=2$

$k=2$

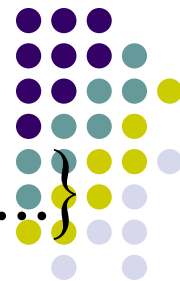
$k=2+2=4$



例 计算 $f_1(k) * f_2(k)$

$$f_1(k) = \{..., 0, 3, 0, 2, 4, 0, ...\}, f_2(k) = \{..., 0, -2, 1, 5, 0, ...\}$$

$k \uparrow = 0$ $k \uparrow = 0$



$$y(k) = \{0, -6, 3, 11, -6, 14, 20, 0\}$$

$k \uparrow = 0$

例 计算 $y(k) = f_1(k) * f_2(k)$ $y(-1)$,

$$f_1(k) = \{0, 1, 2, 0, 3, 0\}, f_2(k) = \{0, -4, 2, 6, 0\}$$

$k \uparrow = 0$ $k \uparrow = 0$

$$y(k) = \{0, -4, -6, 10, 0, 6, 18, 0\}$$

$k \uparrow = 0$

3.4 离散系统时域分析 (§ 3.1)



线性常系数差分方程



模拟框图

(卷积和运算)

$$\begin{cases} \text{经典解法} & y_h(k) + y_p(k) \\ \text{时域解法} & y_{zs}(k) + y_{zi}(k) \end{cases}$$

$$\begin{cases} y_{zi}(k) = \sum_i C_i \lambda_i^k \\ \text{n个与 } f(k) \text{ 无关的初始值} \end{cases}$$

$$\begin{cases} y_{zs}(k) = f(k) * h(k) \\ h(k) \text{ 的求取} & \text{迭代法求初值} \end{cases}$$

一. 零输入响应 $y_{zi}(k)$



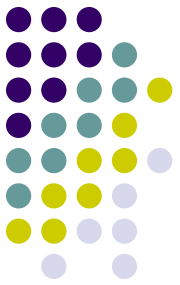
$$y_{zi}(k) + a_1 y_{zi}(k-1) + a_0 y_{zi}(k-2) = 0$$

$$\Rightarrow \lambda^2 + a_1 \lambda + a_0 \lambda = 0 \Rightarrow \lambda_1, \lambda_2$$

$$\Rightarrow \begin{cases} y_{zi}(k) = C_1 (\lambda_1)^k + C_2 (\lambda_2)^k & k \geq 0 \\ y_{zi}(0), y_{zi}(1) \end{cases} *$$

对比: $y_{zi}''(t) + a_1 y_{zi}'(t) + a_0 y_{zi}(t) = 0$

$$\begin{cases} y_{zi}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ y_{zi}(0), y_{zi}'(0) \end{cases}$$



例 3.1-4 求零输入响应:

$$\begin{cases} y(k) + 3y(k-1) + 2y(k-2) = f(k); f(k) = 0, k \geq 0 \\ y(-1) = 0, y(-2) = \frac{1}{2} \end{cases}$$

解:

$$y(-1) = 0, y(-2) = \frac{1}{2} \Rightarrow y_{zi}(0) = -1, y_{zi}(1) = 3$$

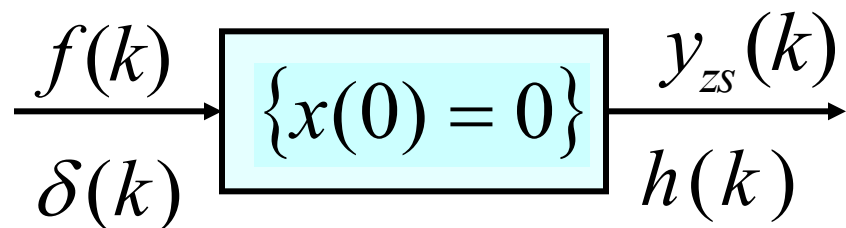
$$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda_1 = -1, \lambda_2 = -2$$

$$\begin{cases} y_{zi}(k) = C_1(-1)^k + C_2(-2)^k \\ y(-1) = y_{zi}(-1) = 0, y(-2) = y_{zi}(-2) = \frac{1}{2} \end{cases}$$

$$\Rightarrow \begin{cases} C_1 + C_2 = -1 \\ -C_1 - 2C_2 = 3 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = -2 \end{cases}$$

$$y_{zi}(k) = (-1)^k - 2(-2)^k, k \geq 0$$

二. 零状态响应 $y_{zs}(k)$



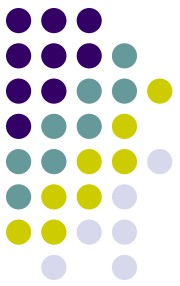
$$\delta(k) \longrightarrow h(k)$$

$$\delta(k-i) \longrightarrow h(k-i)$$

$$f(i)\delta(k-i) \longrightarrow f(i)h(k-i)$$

$$\sum_{i=-\infty}^{\infty} f(i)\delta(k-i) \longrightarrow \sum_{i=-\infty}^{\infty} f(i)h(k-i)$$

$$f(k) \longrightarrow y_{zs}(k) = \sum_{i=-\infty}^{\infty} f(i)h(k-i) = f(k) * h(k)$$



例 求 $y_{zs}(k)$ 已知: $h(k) = (0.5)^k \varepsilon(k)$,
 $f(k) = \varepsilon(k) - \varepsilon(k-5)$

解: $y_{zs}(k) = h(k) * f(k)$

$$= (0.5)^k \varepsilon(k) * [\varepsilon(k) - \varepsilon(k-5)]$$

$$\begin{aligned} (0.5)^k \varepsilon(k) * \varepsilon(k) &= \left[\sum_{i=0}^k (0.5)^i \right] \cdot \varepsilon(k) \\ &= 2[1 - (0.5)^{k+1}] \varepsilon(k) \end{aligned}$$

$$y_{zs}(k) = 2[1 - (0.5)^{k+1}] \varepsilon(k) - 2[1 - (0.5)^{k-4}] \varepsilon(k-5)$$

$$= \begin{cases} 0, & \dots\dots\dots k < 0 \\ 2 - (0.5)^k, & \dots\dots\dots 0 \leq k \leq 4 \\ 31(0.5)^k, & \dots\dots\dots k \geq 5 \end{cases}$$

例 3.1-5 求 $y_{zs}(k)$ 已知:



$$y(k) + 3y(k-1) + 2y(k-2) = f(k), \quad f(k) = 2^k \varepsilon(k)$$

解: $h(k) + 3h(k-1) + 2h(k-2) = \delta(k)$

$$h(k) = [C_1(-1)^k + C_2(-2)^k] \varepsilon(k)$$

$$h(0) = 1 \quad h(1) = -3$$

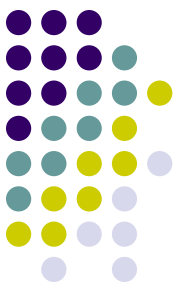
$$h(k) = [-(-1)^k + 2(-2)^k] \varepsilon(k)$$

$$y_{zs}(k) = f(k) * h(k)$$

$$\begin{aligned} & a^k \varepsilon(k) * b^k \varepsilon(k) \\ &= \frac{a^{k+1} - b^{k+1}}{a - b} \varepsilon(k) \end{aligned}$$

$$= [-(-1)^k + 2(-2)^k] \varepsilon(k) * 2^k \varepsilon(k)$$

$$= \left[-\frac{1}{3}(-1)^k + (-2)^k + \frac{1}{3}(2)^k \right] \varepsilon(k)$$



三. 全响应 $y(k) = y_{zs}(k) + y_{zi}(k)$

例 求全响应 $y(k), k \geq 0$, 已知:

$$\begin{cases} y(k) - 0.7y(k-1) + 0.12y(k-2) = 2f(k) - f(k-1) \\ y_{zi}(0) = 8, y_{zi}(1) = 3, f(k) = (0.2)^k \varepsilon(k) \end{cases}$$

解: (1) $\lambda^2 - 0.7\lambda + 0.12 = 0 \rightarrow \lambda_1 = 0.3, \lambda_2 = 0.4$

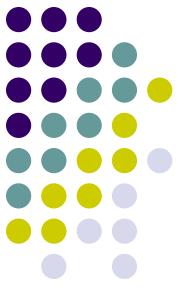
$$\begin{cases} y_{zi}(k) = C_1(0.3)^k + C_2(0.4)^k \\ y_{zi}(0) = 8, y_{zi}(1) = 3 \end{cases} \Rightarrow \begin{cases} C_1 = 2 \\ C_2 = 6 \end{cases}$$

$$y_{zi}(k) = 2(0.3)^k + 6(0.4)^k \quad k \geq 0$$

$$(2) \quad h_1(k) - 0.7h_1(k-1) + 0.12h_1(k-2) = \delta(k)$$

$$h_1(k) = \left[A_1(0.3)^k + A_2(0.4)^k \right] \varepsilon(k)$$

$$h_1(0) = 1, h_1(1) = 0.7$$



$$y(k) - 0.7y(k-1) + 0.12y(k-2) = f(k)$$

$$h_1(k) = [4(0.4)^k - 3(0.3)^k] \varepsilon(k)$$

$$y(k) - 0.7y(k-1) + 0.12y(k-2) = 2f(k) - f(k-1)$$

$$\begin{aligned} h(k) &= 2h_1(k) - h_1(k-1) \\ &= 2[4(0.4)^{\cancel{k}} - 3(0.3)^{\cancel{k}}] \varepsilon(\cancel{k}) - [4(0.4)^{\cancel{k-1}} - 3(0.3)^{\cancel{k-1}}] \varepsilon(\cancel{k-1}) \\ &= 2\delta(k) + [4(0.3)^k - 2(0.4)^k] \varepsilon(k-1) \\ &= [4(0.3)^k - 2(0.4)^k] \varepsilon(k) \end{aligned}$$

$$\begin{aligned} (3) \quad y_{zs}(k) &= f(k) * h(k) \quad y_{zi}(k) = 2(0.3)^k + 6(0.4)^k \\ &= [12(0.3)^k - 4(0.4)^k - 6(0.2)^k] \varepsilon(k) \end{aligned}$$

$$\begin{aligned} (4) \quad y(k) &= y_{zs}(k) + y_{zi}(k) \\ &= 2[7(0.3)^k + (0.4)^k - 0.3(0.2)^k], k \geq 0 \end{aligned}$$